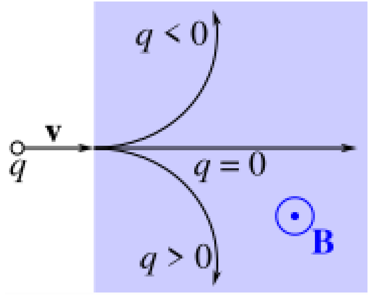
가속기전자석 Part-I
자석용 빔물리기초, 자석 설계

2025. Jan. 13

Pohang Accelerator Laboratory / POSTECH
Div. of Advanced Nuclear Engineering / POSTECH
Garam HAHN

Lorentz Equation

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$$



$$q \cdot 3 \times 10^8 \left(\frac{\mathrm{m}}{\mathrm{s}}\right) \cdot 1\ (\mathrm{T}) = q \cdot E_{1\mathrm{T}\ eq.}$$

m/s * T / (MV/m) = 1e-6

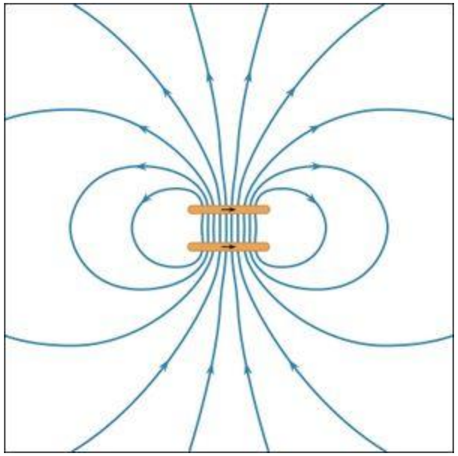
$$E_{1T\ eq.} = 3 \times 10^2\ \mathrm{MV/m}$$

2 cm 간격에 전위차 3000 kV ??

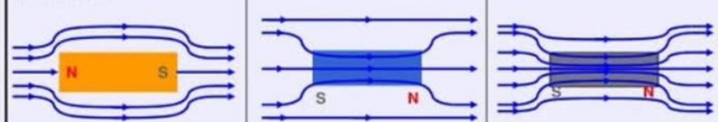
알미늄/구리 등의 polishing된 메탈 표면에서 방전이 일어나는 표면 전기장 세기 15 MV/m

Biot–Savart law

$$d\vec{\mathbf{B}}(\vec{\mathbf{r}}) = \frac{\mu_0}{4\pi} I d\vec{\mathbf{l}} \times \hat{\mathbf{r}}$$



DIA	PARA	FERRO
1. Diamagnetic substances are those substances which are feebly repelled by a magnet. Eg. Antimony, Bismuth, Copper, Gold, Silver, Quartz, Mercury, Alcohol, water, Hydrogen, Air, Argon, etc.	Paramagnetic substances are those substances which are feebly attracted by a magnet. Eg. Aluminium, Chromium, Alkali and Alkaline earth metals, Platinum, Oxygen, etc.	Ferromagnetic substances are those substances which are strongly attracted by a magnet. Eg. Iron, Cobalt, Nickel, Gadolinium, Dysprosium, etc.
2. When placed in magnetic field, the lines of force tend to avoid the substance.	The lines of force prefer to pass through the substance rather than air.	The lines of force tend to crowd into the specimen.



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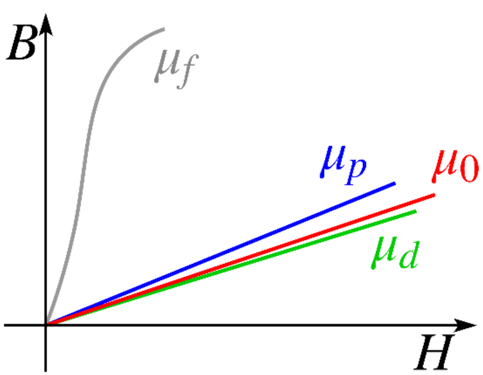
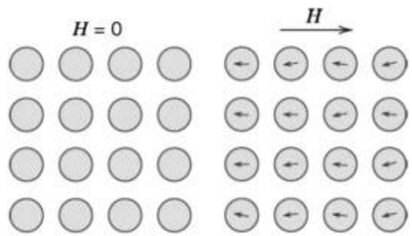
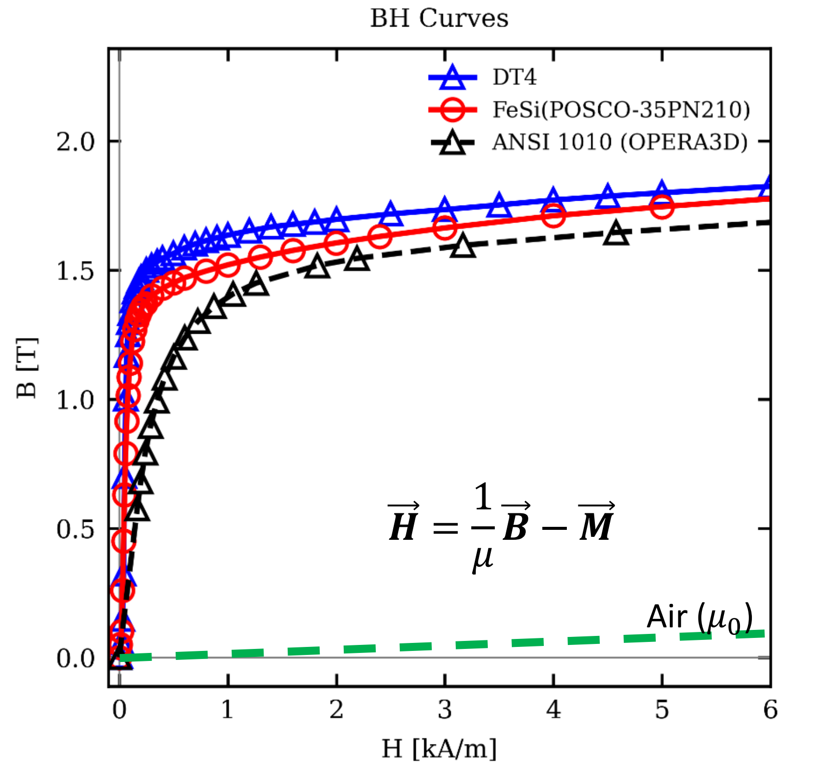






Orbit Stability

John J Livingood - Principles of Cyclic Particle Accelerators (1961)

결국 자석이 하는 일은

(1) 방향 전환
(2) 보존장 형성

(1) 특히,
cyclic 빔 궤도를 갖는 가속기에서

빔이 equilibrium orbit 을 형성하도록
닫힌 궤도를 만드는 일

(2)
궤도의 중심으로 보존력을 만들어
멀리 벗어나지 않도록 하는 일

→ Oscillation!!

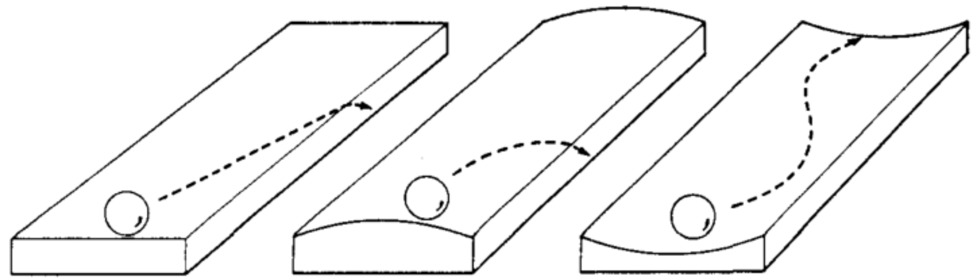


Fig. 2-1. Bowling alley analogy of systems of forces which produce neutral, unstable, or stable orbits.

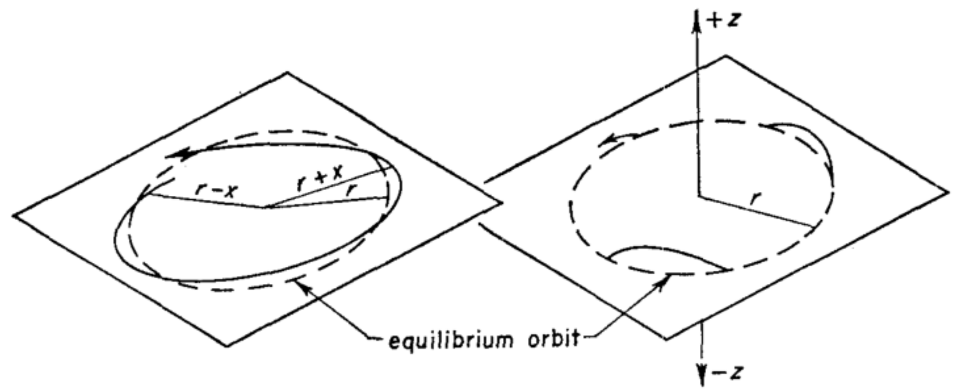


Fig. 2-2. Schematic representation of betatron oscillations about a circular equilibrium orbit. Radial motion is shown occurring on the midplane of the magnetic field, while axial motion is shown on a cylindrical surface. In actuality, both motions occur simultaneously. With weak-focusing machines, there is always less than one oscillation of either kind per turn.

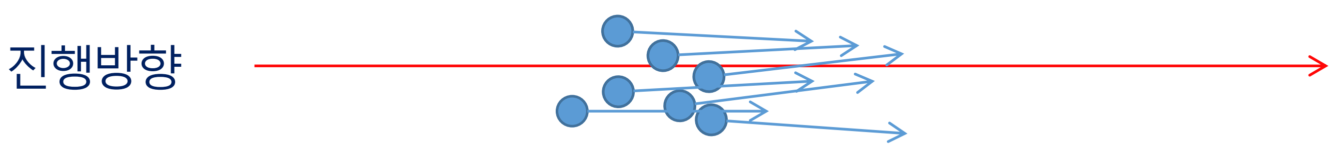
빔 역학 기초

- 힘의 전달
  - 빔의 진행 방향을 바꿀땐 주로 자기장을 이용한다. (가끔 전기장도 이용한다.)
  - 빔에 에너지를 변화시킬땐 <u>단지 전기장 만 이용한다</u>.

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$$

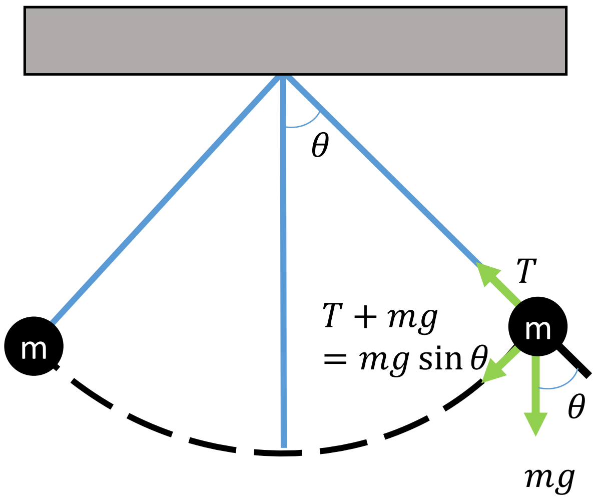
$$\vec{F}_{\perp} = q\vec{E}_{\perp} + q\vec{v}_{||} \times \vec{B}_{\perp}, \qquad \vec{F}_{||} = q\vec{E}_{||} \cancel{+ q\vec{v}_{||} \times \vec{B}}$$

- 빔은 같은 전하를 띈 입자들의 다발이며, 일반적으로 놔두면 퍼진다, 입자들은 제각기 다른 방향이기 때문
  - 엄밀히, 하전 입자는 약간씩 다른 방향으로 직진한다 → 놔두면 퍼진다



  - 위 그림과 같이 빔은 공간적인 분포와 각도의 분포를 갖는다

What is the phase space?



$x(t) = Asin(\omega_0 t - \delta)$

$\dot{x}(t) = A\omega_0 cos(\omega_0 t - \delta)$

Equation of motion along $\theta - direction$

$$F_\theta = m\frac{d}{dt}v_\theta \Rightarrow -mgsin\theta = m\frac{d}{dt}(l\dot{\theta})$$

$$\Rightarrow \ddot{\theta} + \frac{g}{l}sin\theta = 0$$

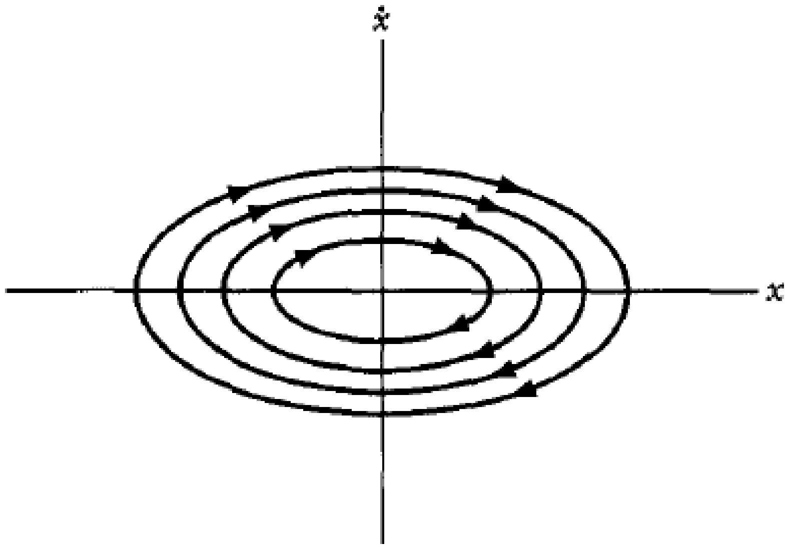
For small θ , $sin\theta \cong \theta$ $\Rightarrow$ $\ddot{\theta} + \frac{g}{l}\theta = 0$

$$\Rightarrow \ddot{\theta} + \omega_0^2\theta = 0$$ where $\omega_0 = \sqrt{\frac{g}{l}}$

: simple harmonic motion

초기 위치와 무관하게 진동수가 동일!

What is the phase space?



서로 다른 초기조건 ➡ 다른 운동 경로

각각의 경로는 **입자의 변화 기록**을 담고 있다.

➡ **Phase diagram** : totality of **all** possible phase **path**

➡ 각각의 경로는 **전체 에너지에 대응**된다.

서로 다른 경로 사이에 **교차할 수 없다**!

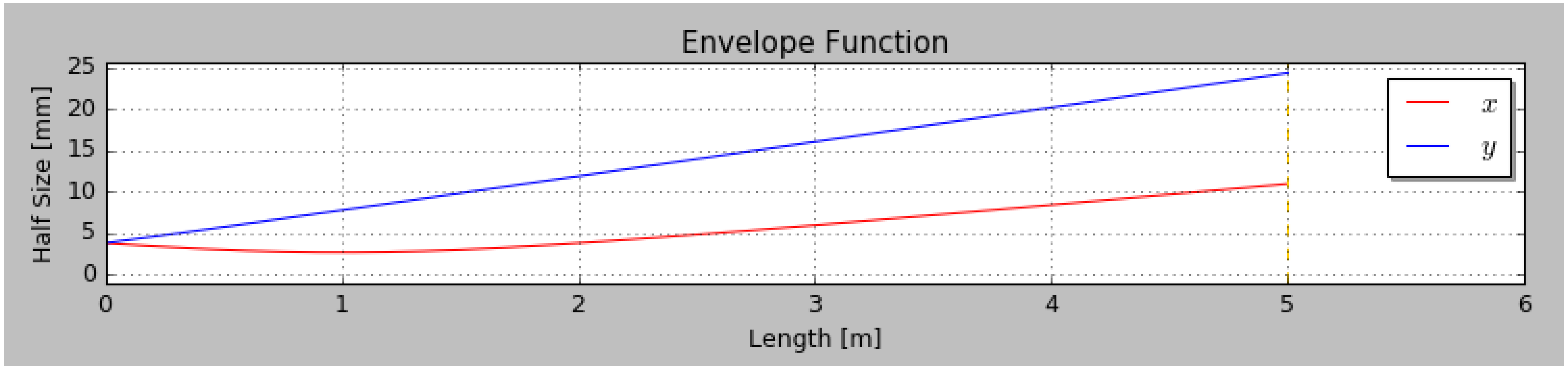
Notice) 위상공간에서 **운동방향은 시계방향**이다.

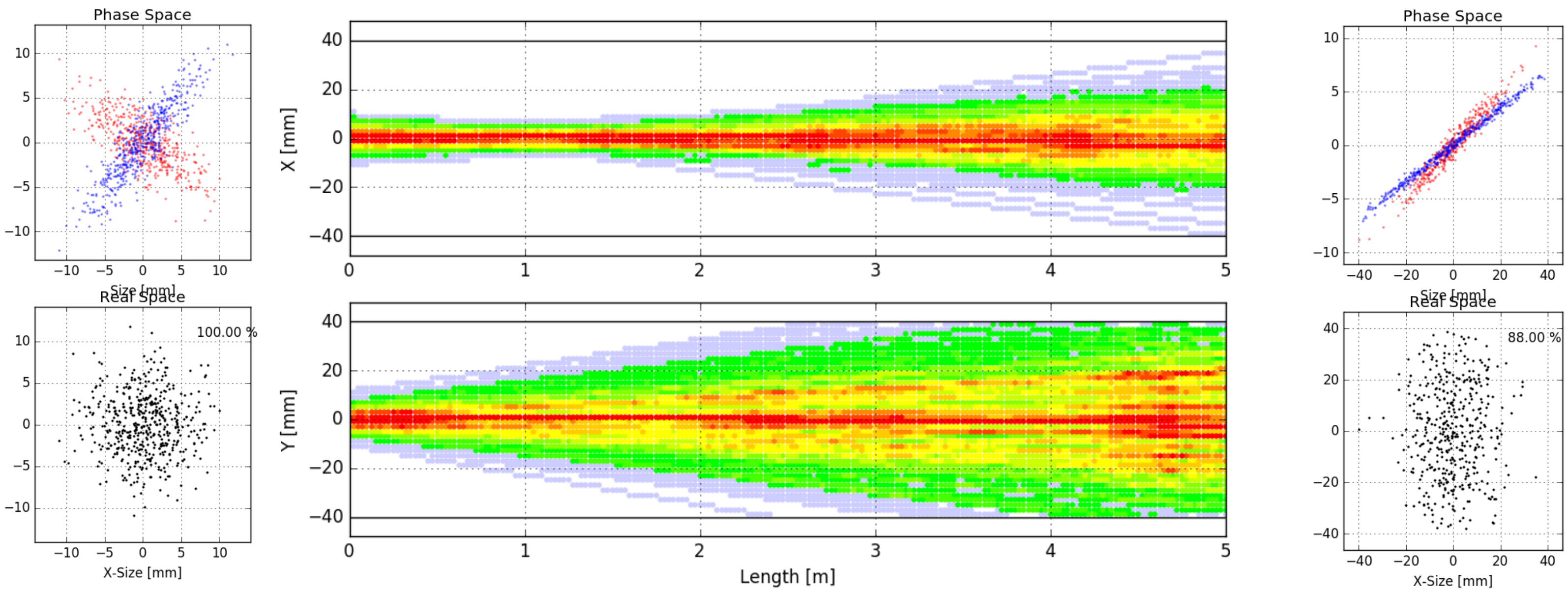
왜냐하면, $x > 0$일 경우에는 속도 $\dot{x}$ 가 항상 감소하고,
$x < 0$ 일 경우에는 속도 $\dot{x}$가 항상 증가함.

수많은 입자의 운동을 통계적으로 표현할 방법이 생김!

예) 에미턴스: 위상공간에서 넓이
빔크기: 한 축으로 차지한 면적

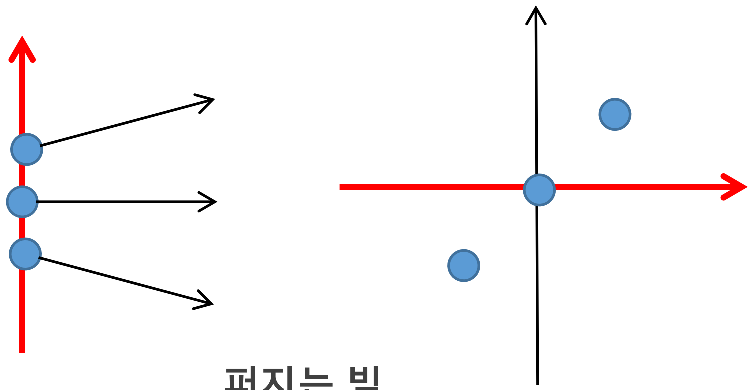
Drift의 양상


Envelope Function
Half Size [mm]
Length [m]
x
y

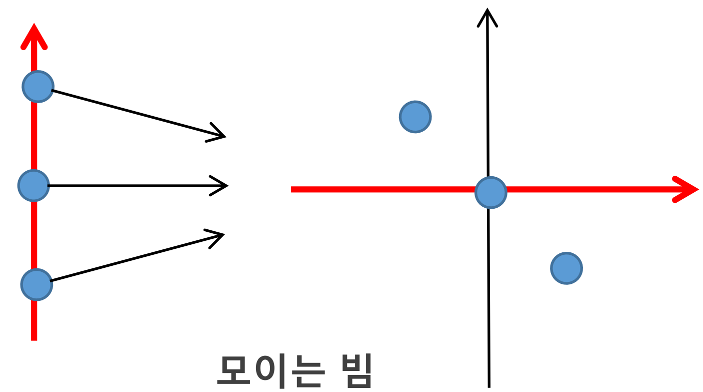

Phase Space
Size [mm]
Real Space
100.00 %
X-Size [mm]
X [mm]
Y [mm]
Length [m]
Phase Space
Size [mm]
Real Space
88.00 %
X-Size [mm]

Drift

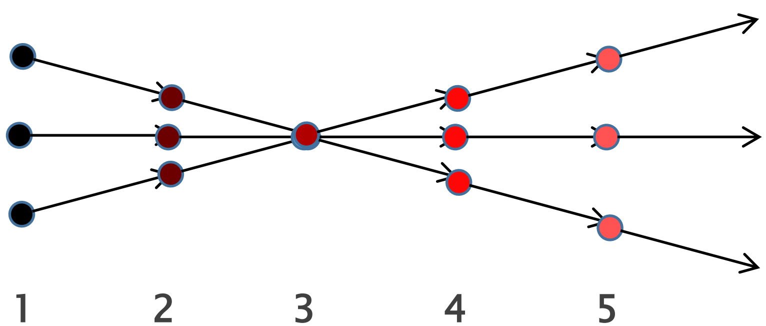
- Phase space 이해하기



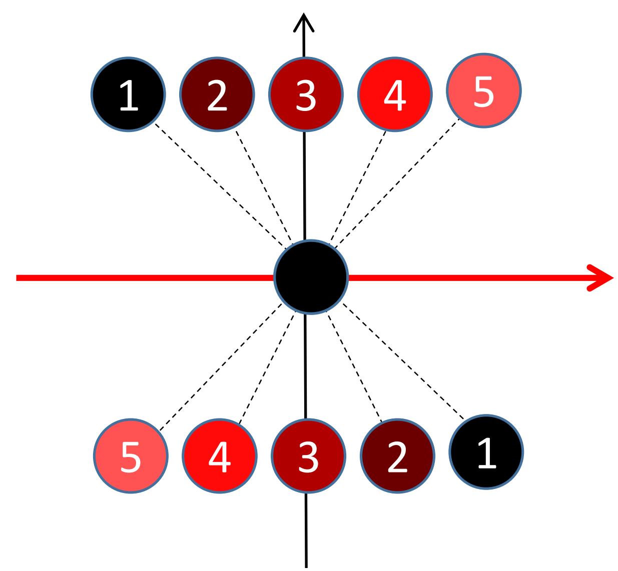
퍼지는 빔



모이는 빔

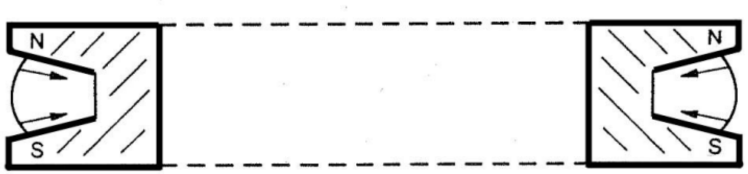


모이는 빔 → 모인 빔 → 퍼지는 빔



빔 집속 개념 탄생

- 초창기 cyclotron이나 synchrotron 에서는 dipole 만 썼음
  - 이때 빔이 퍼지는 걸 막기 위해 focusing 이란게 필요하다고 판단
  - 자기장을 $B_y = B_0/r^n$ 형태로 만들면 EQ orbit 근방에서 보존력이 만들어 짐 알게됨
  - 이 조건을 방정식으로 만들어 풀면 아래처럼 나오는데, (Living good 책에 나옴)
    x" + (1-n)x = 0, y" + ny = 0
  - 수평방향 튠이 $\sqrt{1-n}$, 수직방향 튠이 $\sqrt{n}$ 으로 나옴 (튠 칠판에 설명)
  - 허수는 발산을 의미 → 튠 자체는 일단 1을 못넘기고... 그 1도 나눠 갖는 구조 (참고로 PLS-II 튠이 2 훌쩍 넘음)
  - 너무 허약한 focusing 이지만, 이거 밖에 없어서 이걸로 가속기 만듦... 다이폴 온리
    에너지가 높으면 밖으로 탈출!, 낮으면 안으로 쳐박!! (에너지 억셉턴스 너무 좁음)



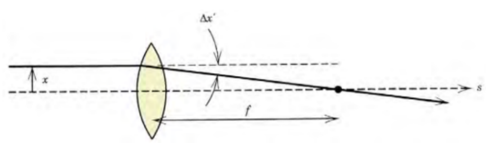
Cross section of weak focusing circular accelerator

US Particle Accelerator School

Strong Focusing, Beam Optics의 탄생

- 만일 Lens를 쓴다면? ➔ energy offset시에도 중심에서 진동!
  - B'이 상수라면, $B_y = \frac{\partial B_y}{\partial x}dx = B'x$ 로 만들고 광학 툴을 그대로 활용 가능!!!

    beam optics 탄생!!
  - 안타깝게 자기장의 성질상, X와 Y의 집속 성분은 반대 x집속-y반집속, vice versa
  - 그럼 둘 다 집속 못하는거 아님? → 아님.

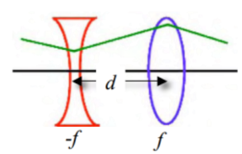


- A lens transforms a ray as

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{out} = \begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix} \begin{pmatrix} x \\ x' \end{pmatrix}_{in}$$

- For a concave lens $f < 0$
- For a drift space of length d

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{out} = \begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ x' \end{pmatrix}_{in}$$



$$\begin{pmatrix} x \\ x' \end{pmatrix}_{out} = \begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix} \begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 1/f & 1 \end{pmatrix} \begin{pmatrix} x \\ x' \end{pmatrix}_{in}$$

$$= \begin{pmatrix} 1 + d/f & d \\ -d/f^2 & 1 - d/f \end{pmatrix} \begin{pmatrix} x \\ x' \end{pmatrix}_{in}$$

For $0 < d << f$, the net effect is focusing with $f_{net} \approx f^2/d > 0$

The same is true if we put the convex lens first

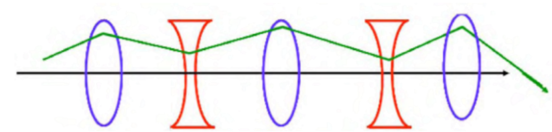
From optics we know that a combination of two lenses, with focal lengths f_1 and f_2 separated by a distance d, has

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

If $f_1 = -f_2$, the net effect is focusing!

∴ A quadrupole doublet is focusing in both planes!

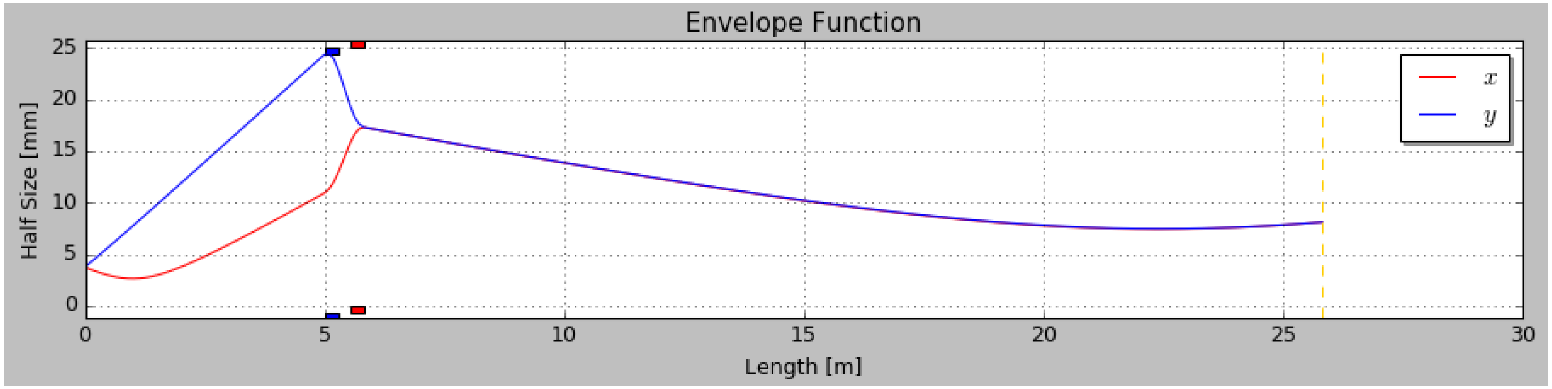
=> Strong focusing by sets of quadrupole doublets with alternating gradient



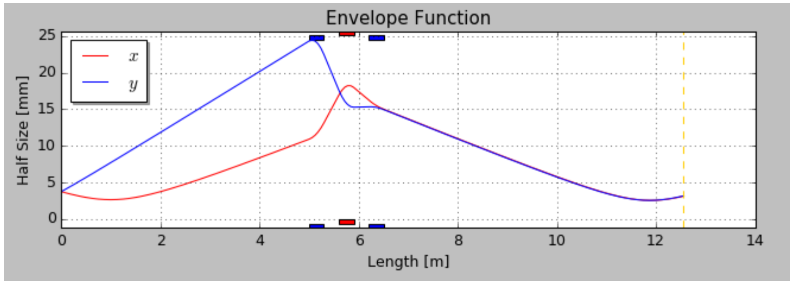
N.B. This is only valid in thin lens approximation

Quadrupole 사용의 예

- 임의 드리프트 뒤에 4극 자석 설치 시
  - 조건 : X, Y 크기를 같게 (단면을 동그랗게) 하고, beam size를 제어함



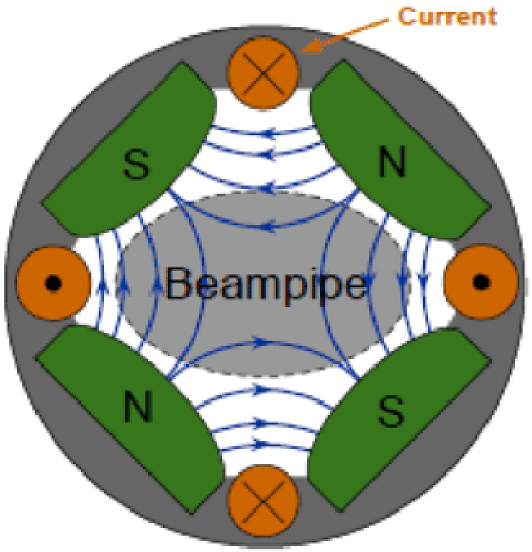
- Triplet은 보다 강한 집속력을 제공하며, 빔 사이즈 조절의 자유도가 훨씬 큼

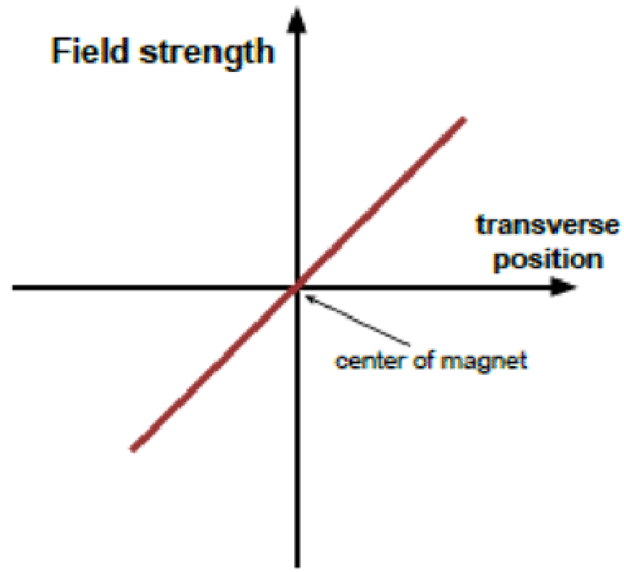


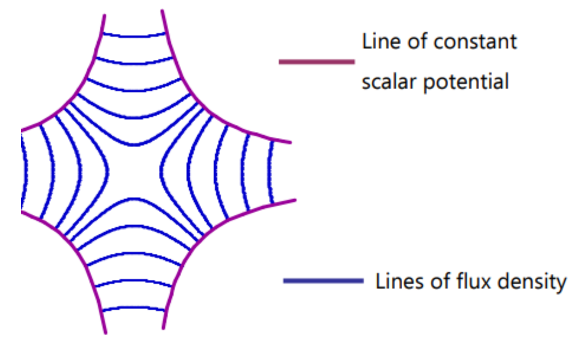
- Iso-center 직전에는 보통 triplet을 사용함
- 설치 공간의 문제 등으로 인해 개수를 줄이거나, 다양한 곳에 분포시켜 사이즈를 조절하는 경우도 있음

Beam manipulation

사극전자석(Quadrupole magnet)





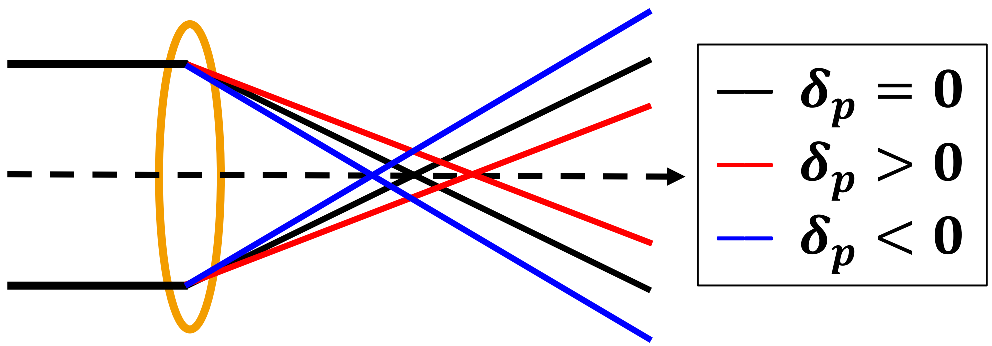


$$B_x = 2(J_2 x + K_2 y), \qquad B_y = 2(K_2 x - J_2 y)$$

$J_2 = 0$ **for 'normal' quadrupole field**

출처: 강릉원주대학교 황지광 교수 강의노트

색수차(Chromaticity)란?



사극전자석에 의한 v_x 변화

$$v_x = \frac{-ev_z}{mv}\int B_y\,ds = \frac{-ev_z}{p}\int B_y\,ds$$

만약 에너지가 달라지면?! $p = p_0(1+\delta)$

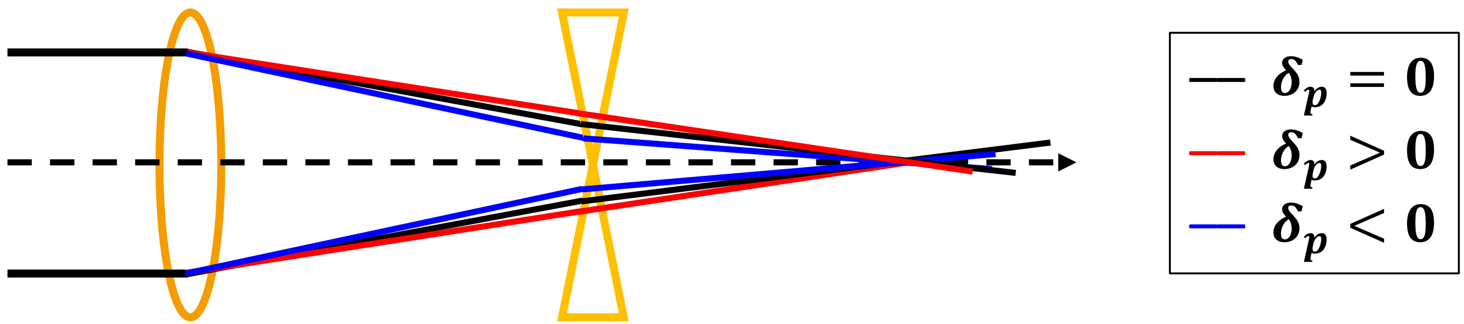
$$v_x = \frac{-ev_z}{p_0(1+\delta_p)}\int 2K_2 x\,ds \approx \frac{-ev_z}{p_0}(1-\delta_p)\int 2K_2 x\,ds = \frac{-ev_z}{p_0}(1-\delta_p)2K_2 xL$$

$$\frac{1}{1+x} \approx 1 - x + \frac{1}{2}x^2 + \cdots$$

출처: 강릉원주대학교 황지광 교수 강의노트

Beam manipulation

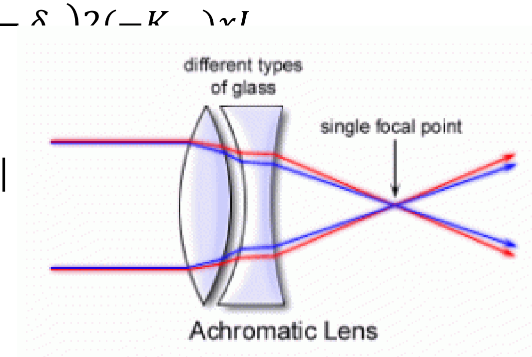
색수차보정 주의사항!



만약 FODO 형태로 자석이 놓여있다면?

$$v_x = \frac{-ev_z}{p_0}(1-\delta_p)2K_{2F}xL + \frac{-ev_z}{p_0}(1-\delta_p)2(-K\ \ )xL$$

서로 상쇄가 가능함! * Achromatic Lens



이러한 이유로 FODO Lattice에는 색수차 보정을 잘하지

예) PAL-XFEL undulator beamline
PAL 3GeV Linac
중이온가속기 선형가속기 등

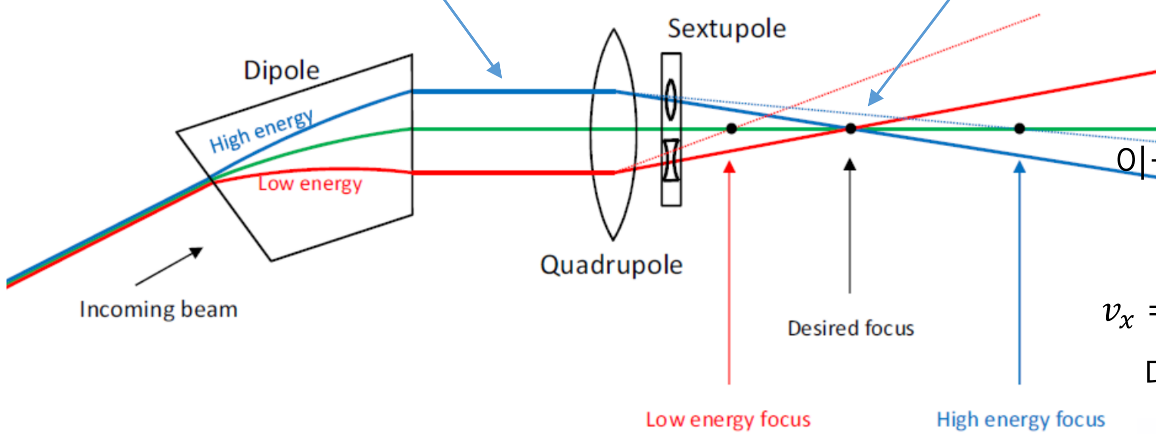
출처: 강릉원주대학교 황지광 교수 강의노트

Beam manipulation

색수차(Chromaticity) 보정을 위해서 필요한 자석은?

이극전자석을 지난 후
에너지에 따른 경로 차
$x = x_0 + \eta\delta_p$

사극전자석에 의한 색수차
$v_x = \frac{-ev_z}{p_0}(1 - \delta_p)2K_2xL$



이극전자석 이후 경로 차로부터
$\delta_E = (x - x_0)/\eta$
을 위 식에 대입하면
$v_x = \frac{-ev_z}{p_0}(1 - (x - x_0)/\eta)2K_2xL$
따라서, $\boldsymbol{v_x \propto x^2}$ **인 효과발생**

$$v_x = \frac{-ev_z}{mv}\int B_y\,ds \;\rightarrow\; B_y \propto x^2$$

인 자석이 필요함!

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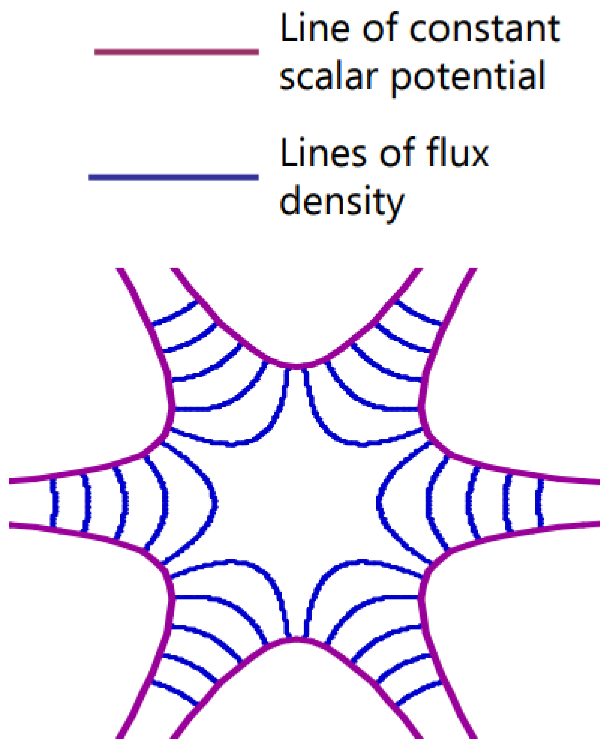
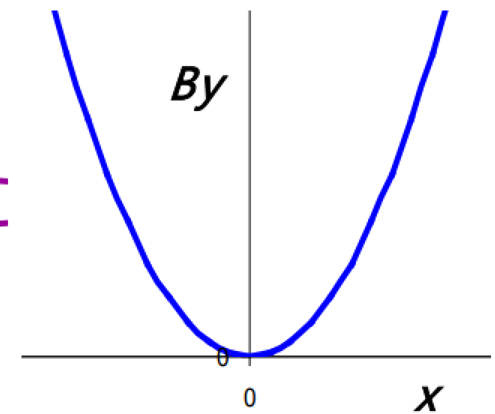
Beam manipulation

육극전자석 (Sextupole magnet)



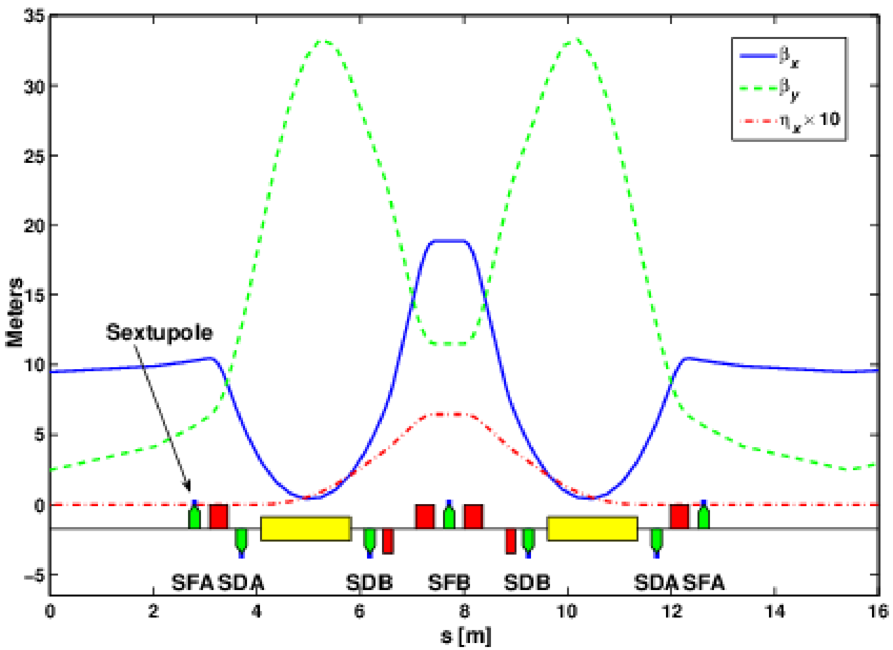


$$B_x = 3(J_3(x^2 - y^2) + 2K_3xy), \qquad B_y = 3(K_3(x^2 - y^2) - 2J_3xy)$$

$J_3 = 0$ for 'normal' sextupole field

출처: 강릉원주대학교 황지광 교수 강의노트

색수차보정 주의사항!



이극전자석 이후 경로 차로부터

$$\delta_E = \frac{x - x_0}{\eta}$$

조건이 만족하는 영역에 육극전자석이 설치되어야
색수차 보정이 가능함.
→ η이 존재하는 영역!

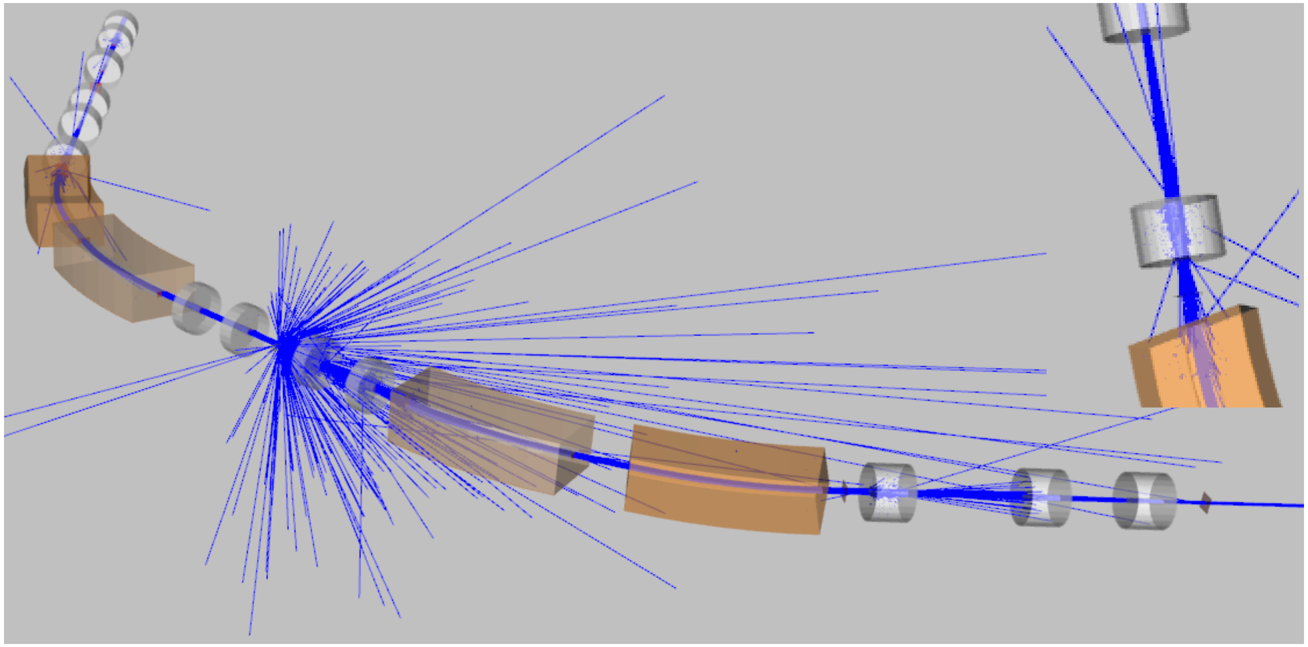
이러한 이유로! Lattice에서 η이 존재하는 영역에
설치된 육극전자석을
Chromatic sextupole이라고도 부름.

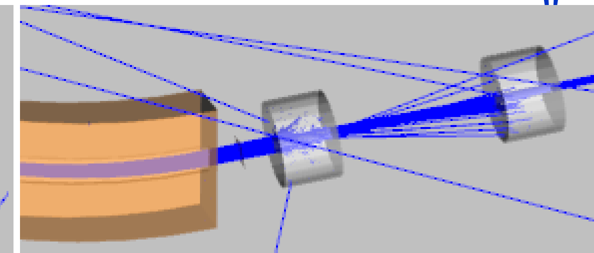
만약 η=0 인 영역에 육극전자석이 있을 경우엔?
색수차에는 변화가 없으며,
Harmonic sextupole이라고 함.

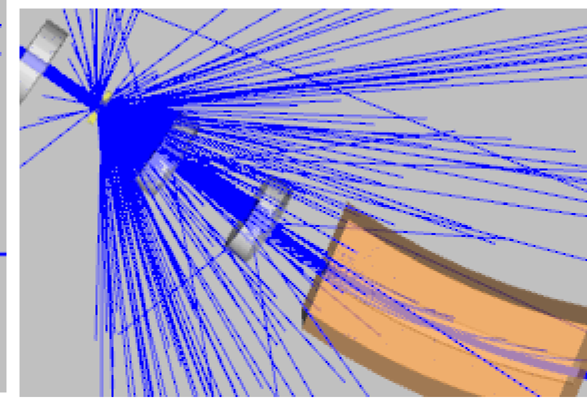
PLS-II 에서는 모든 곳에
η이 존재하므로 구분이 없음.

출처: 강릉원주대학교 황지광 교수 강의노트

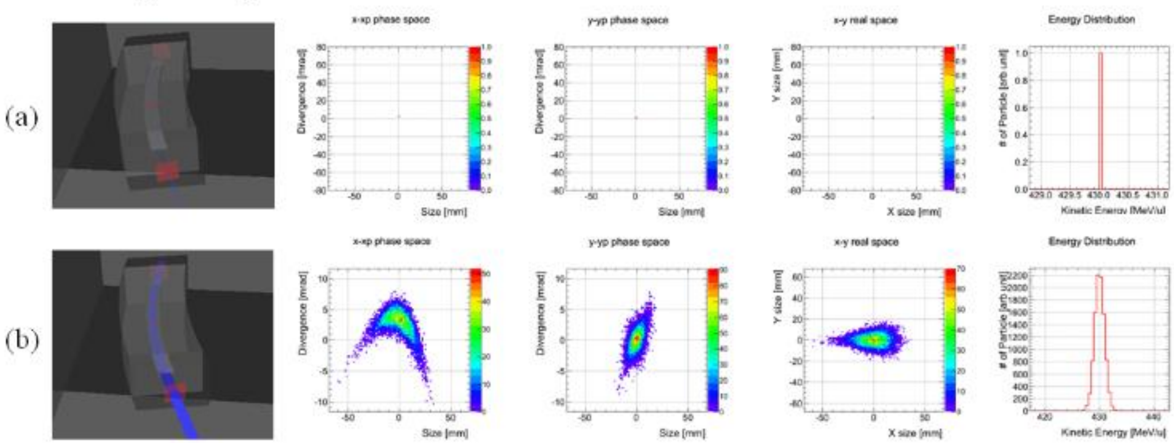
전자석 튜닝이 잘못 되었을 때의 예



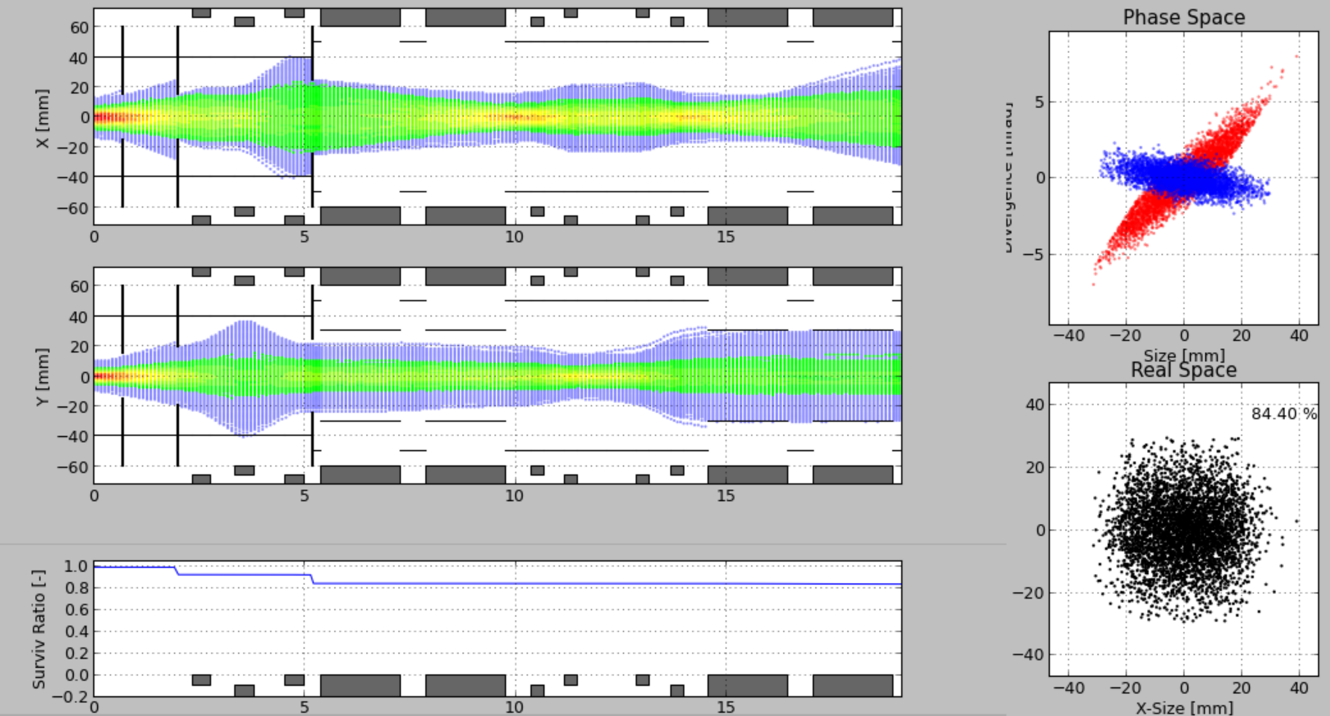




- Rectangular Dipole



- 약 40% 의 빔이 전송 중 유실됨
- Quadrupole 은 Dispersion 제어에 부적합한 필드 오차 포함
- Dipole은 Saturation 영역에 접근 Non-linear 효과가 극대화됨
- 모두 재설계 검토



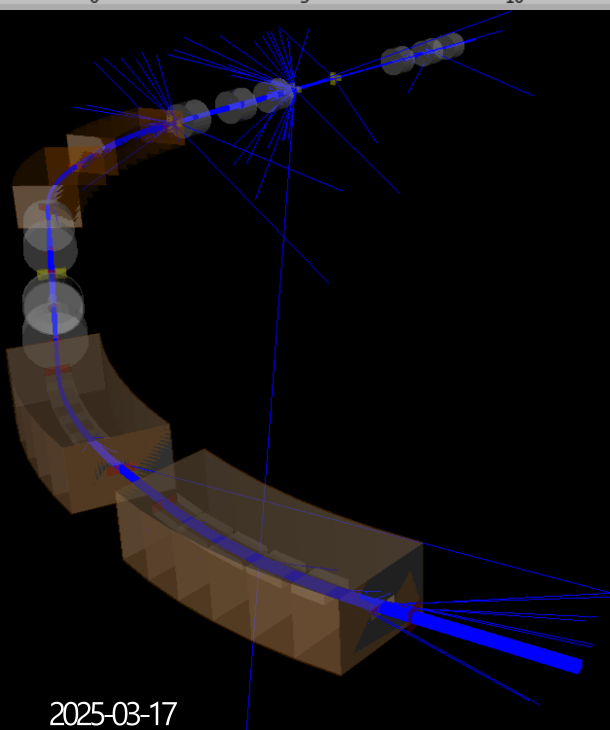
자석 보정 이후,
(1) 추가 빔 손실 제거됨

(2) Optics와 완전 일치함

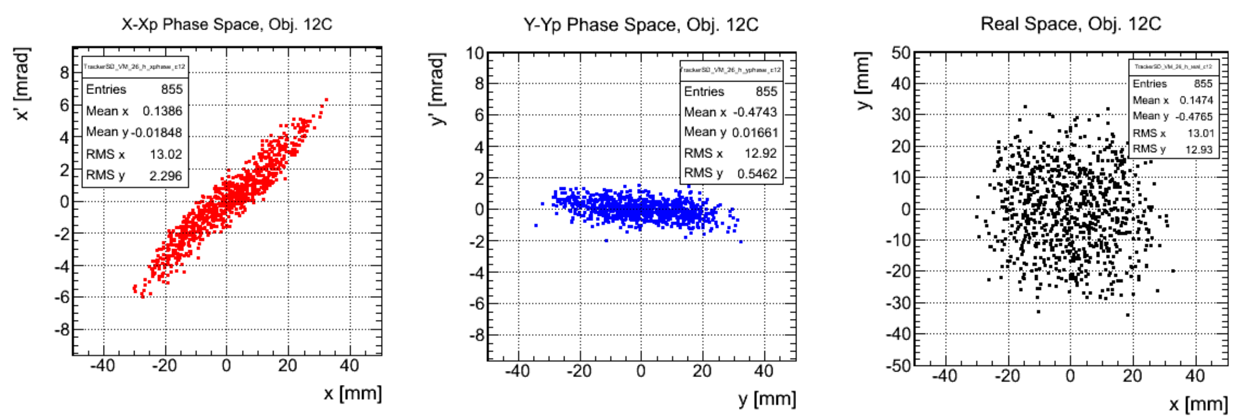
보정된 자석

(1) RB(Bending)
Full Gap Size 증가
6 cm → 7 cm

(2) QP3(Quadrupole)
내경 증가
8 cm → 10 cm

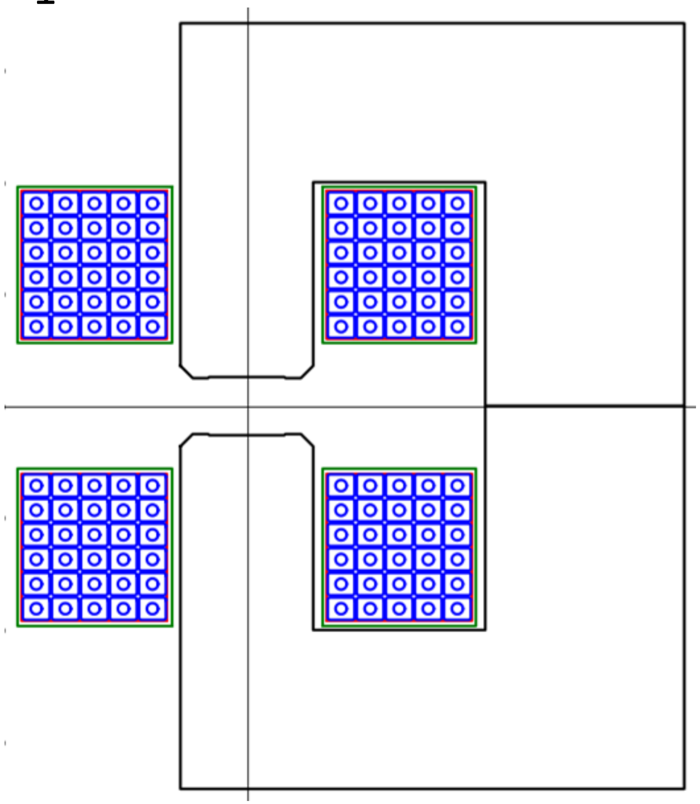


동일위치(Section-3 끝)에서의 3D Tracking 결과

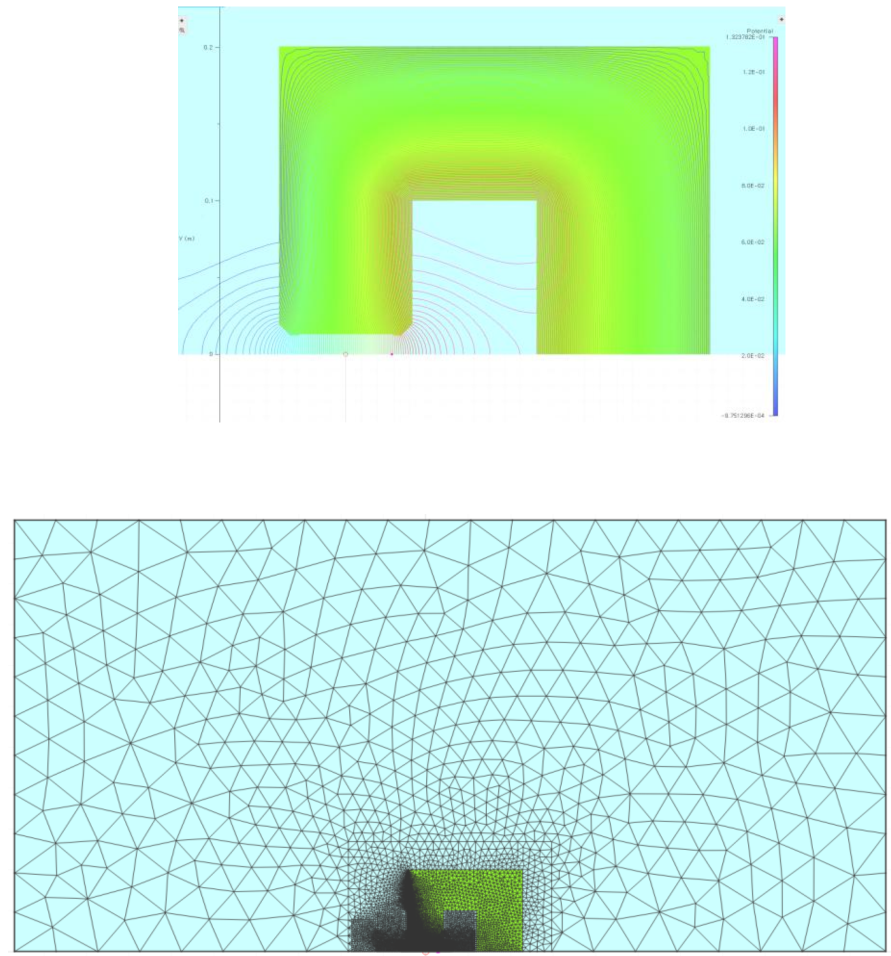


Dipole

1

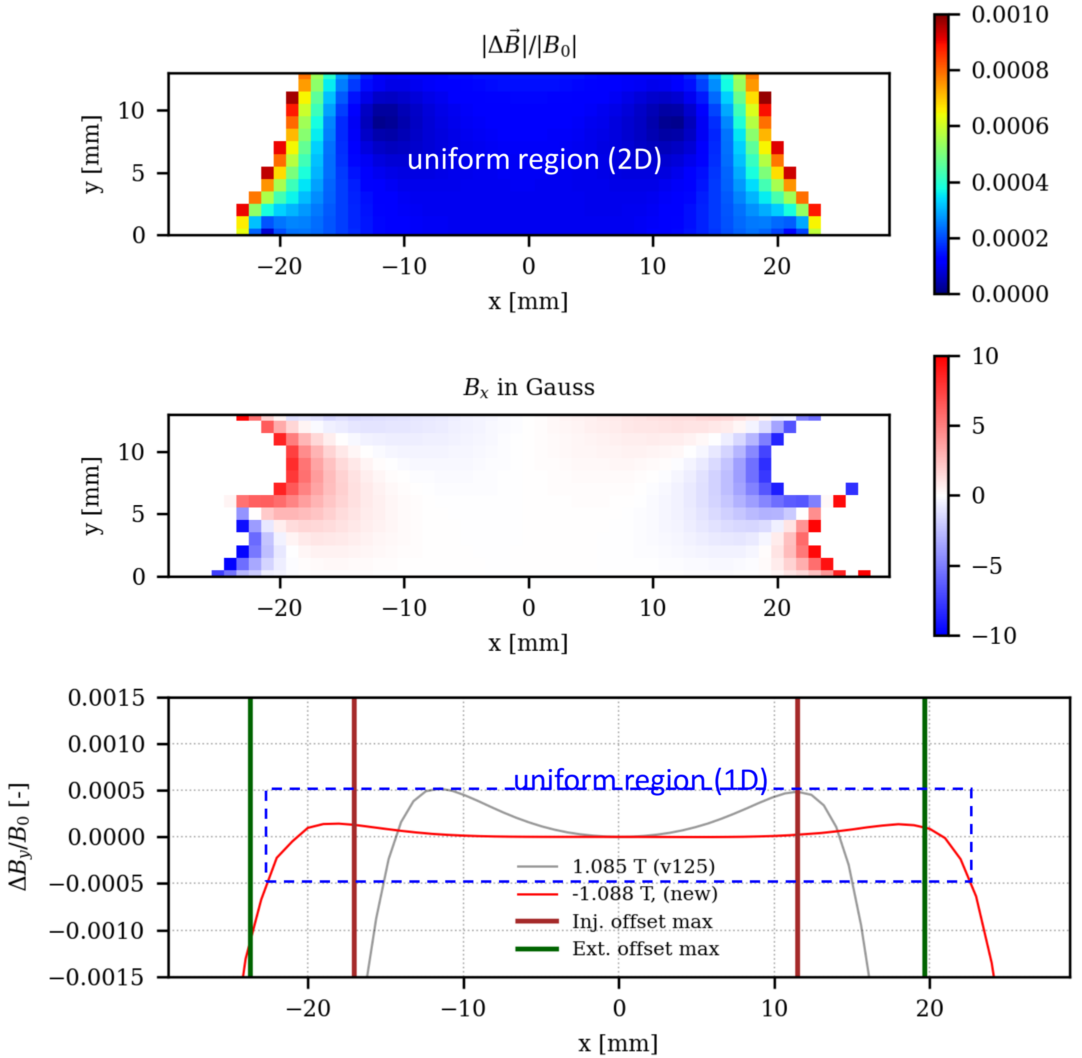


2



Dipole

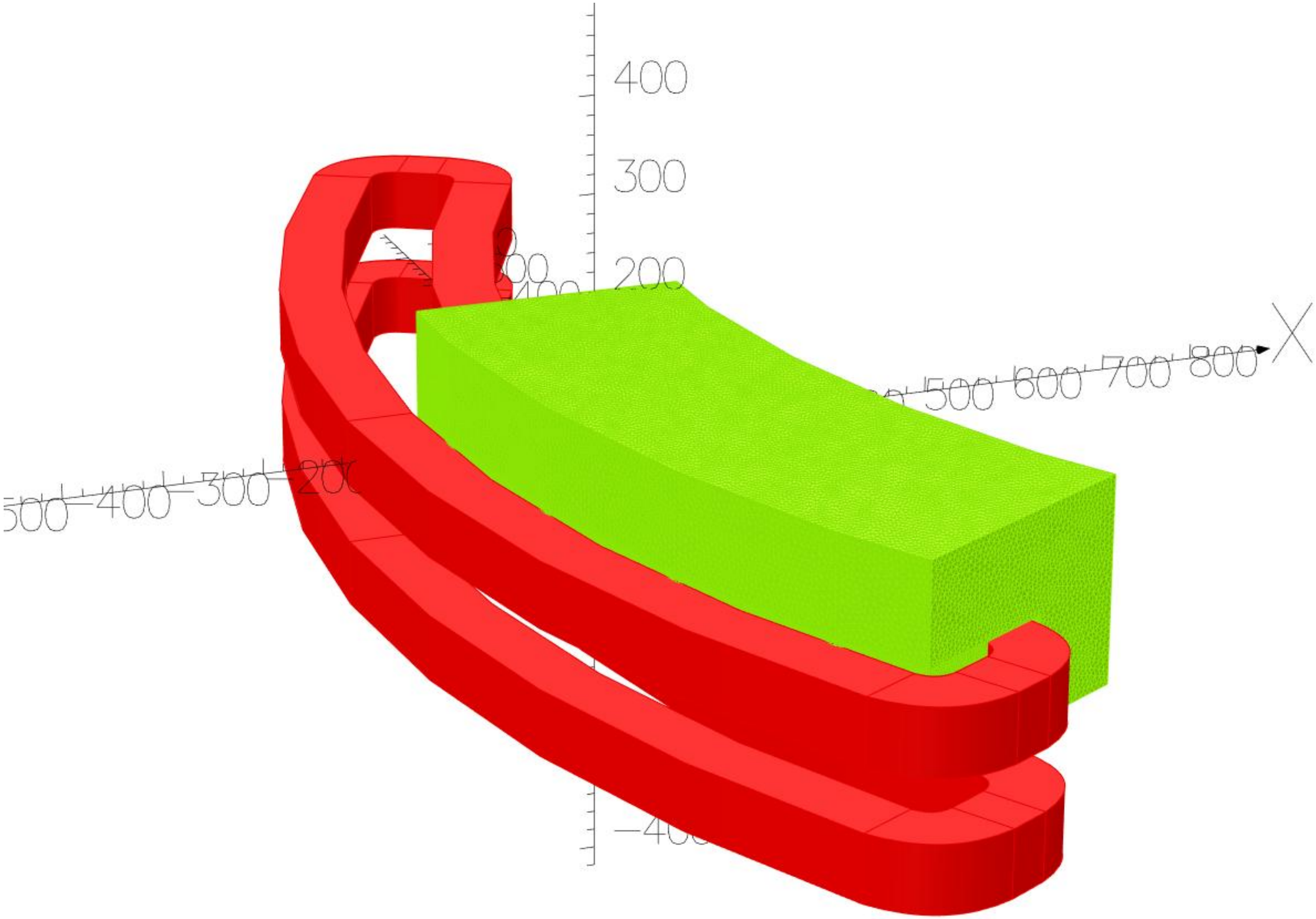
3



$|\Delta\vec{B}|/|B_0|$
uniform region (2D)
y [mm]
x [mm]
0.0010
0.0008
0.0006
0.0004
0.0002
0.0000
B_x in Gauss
10
5
0
−5
−10
uniform region (1D)
$\Delta B_y/B_0$ [-]
1.085 T (v125)
-1.088 T, (new)
Inj. offset max
Ext. offset max

Dipole

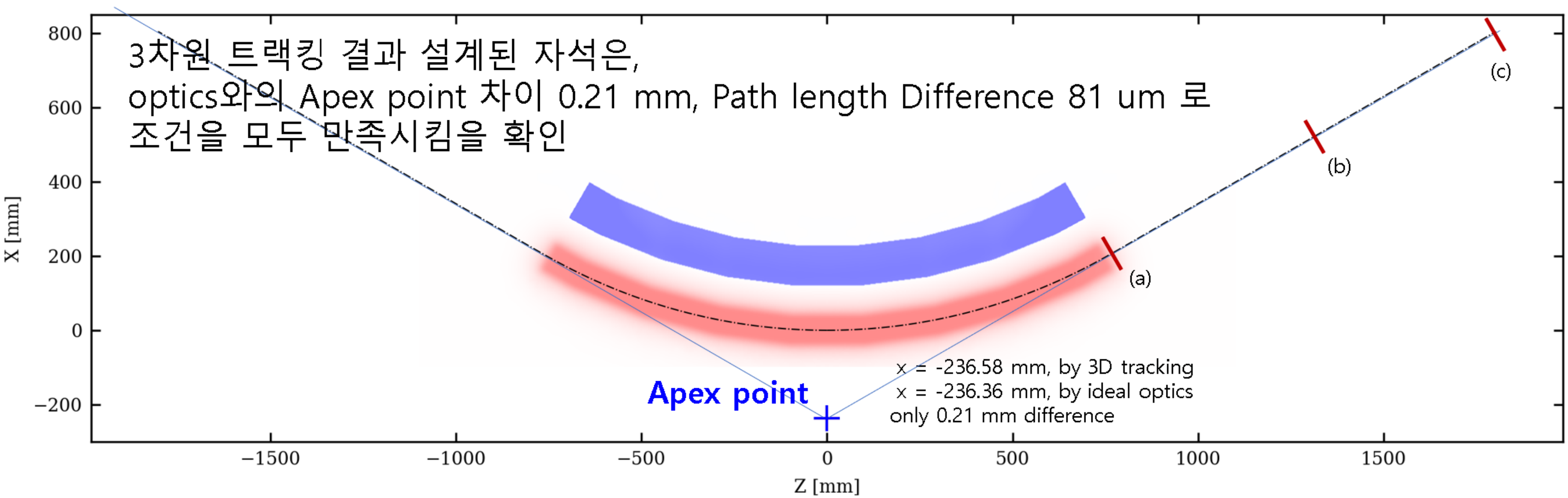
4



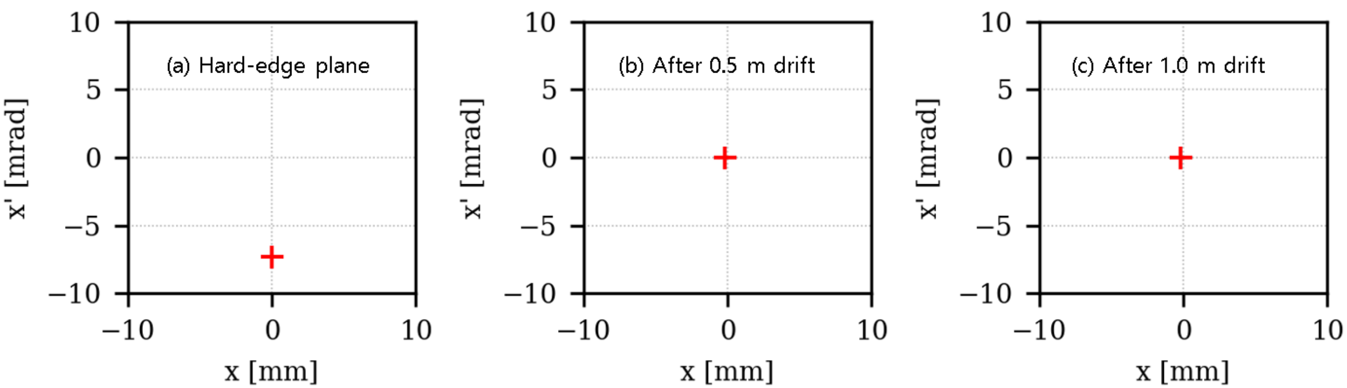
Dipole

5

3차원 트랙킹 결과 설계된 자석은,
optics와의 Apex point 차이 0.21 mm, Path length Difference 81 um 로 조건을 모두 만족시킴을 확인



Reference Orbit Tracing



Path Length Difference

80.957 um / each dipole

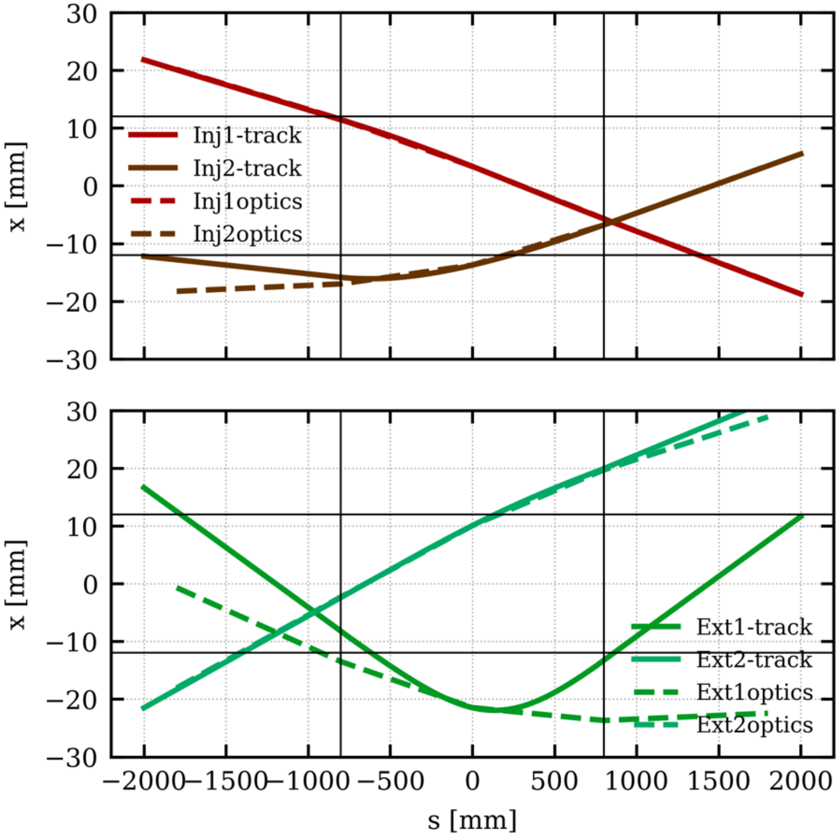
485.742 um / 1-turn revolution

~ 0.29 deg shift in 500 MHz RF phase per 1-turn

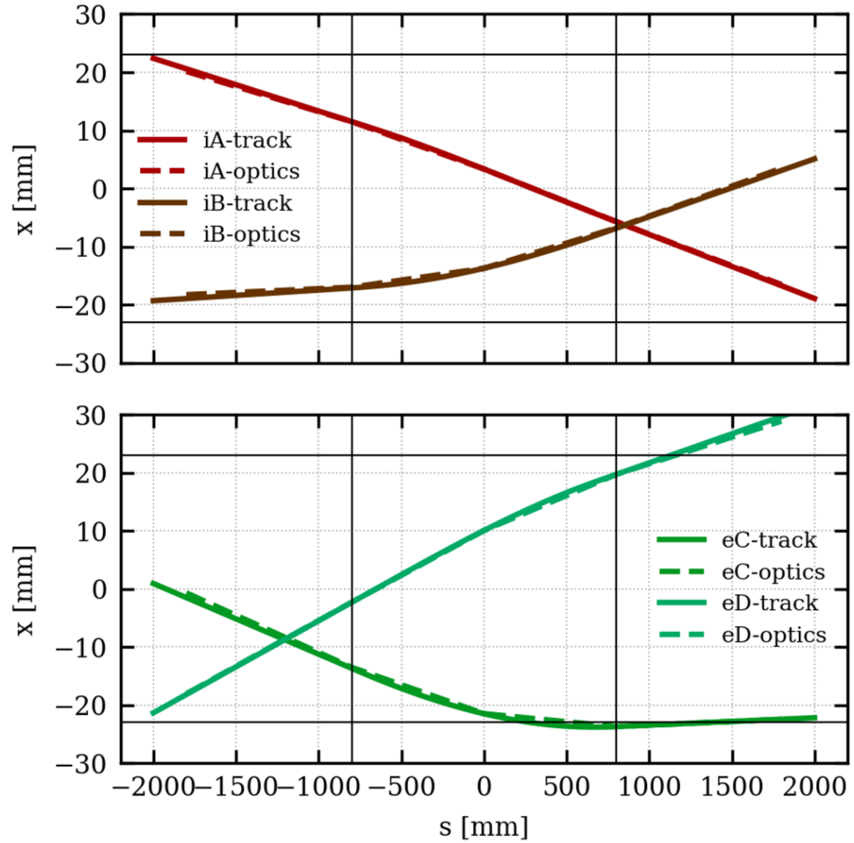
6

새 디자인은, 기존 디자인과 비교, 입출사 문제를 모두 수정했음.

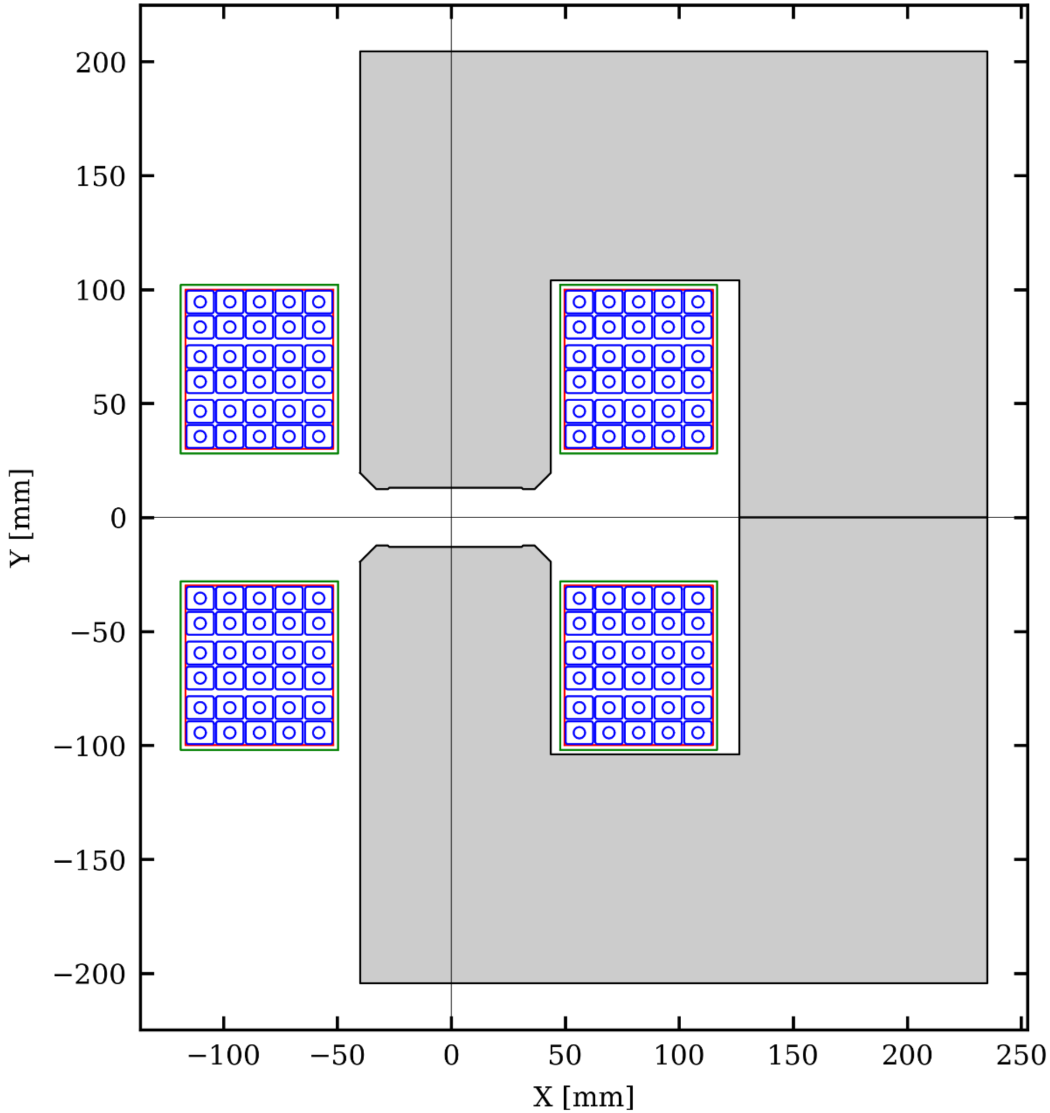
Previous Version



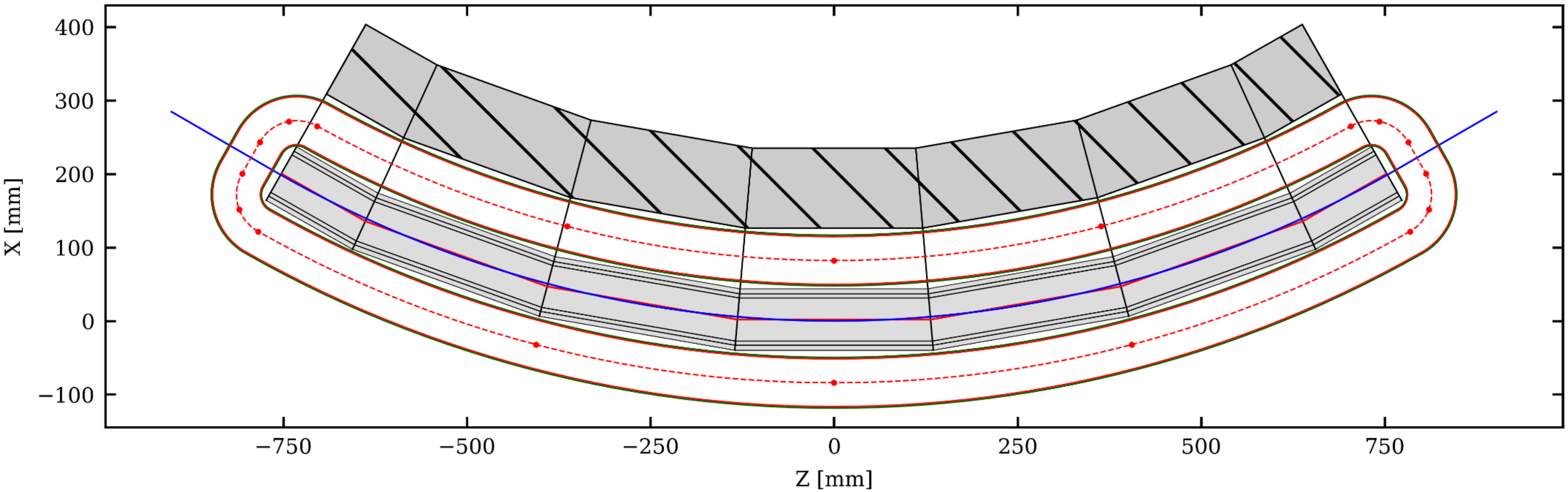
New Version



Dipole

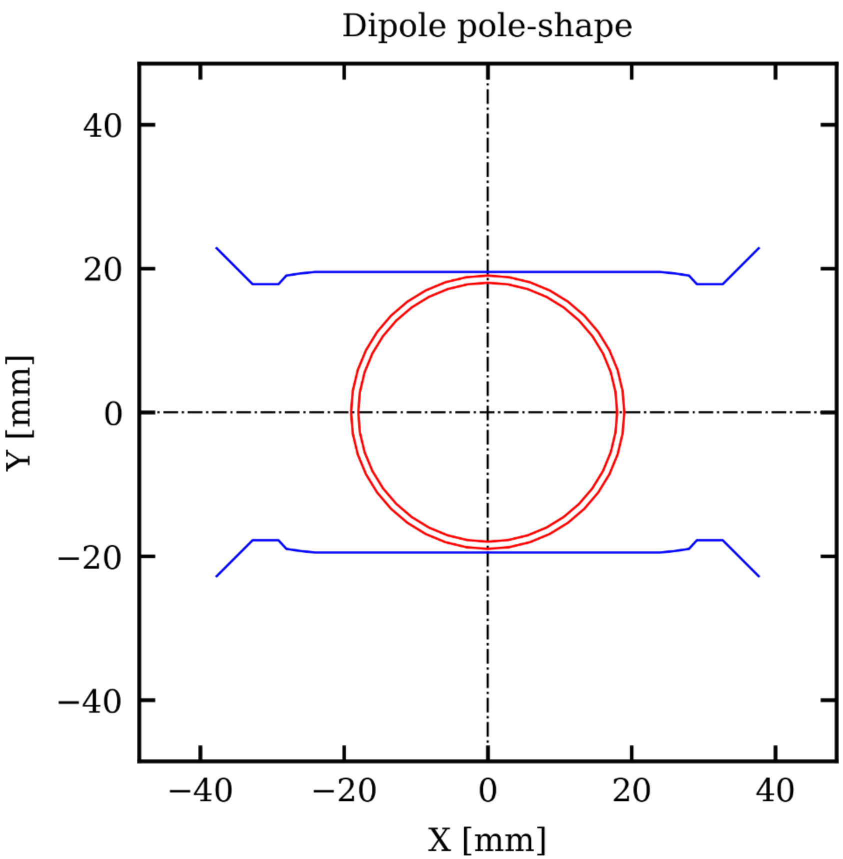


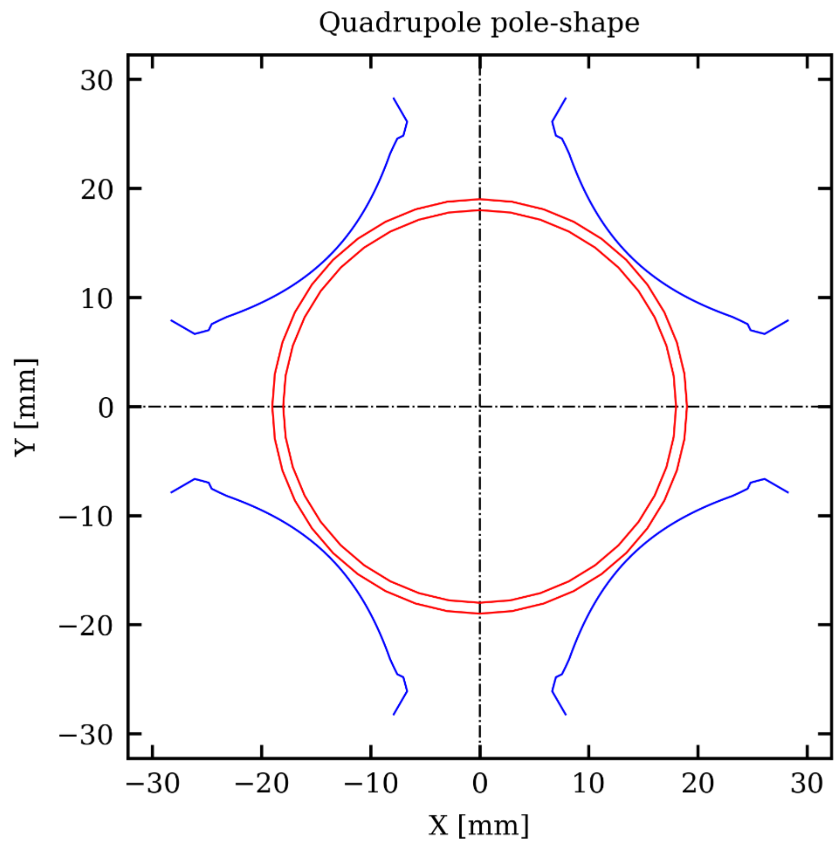
Dipole



Quadrupole

- conformal mapping D → Q





Mapping a Dipole into a Quadrupole

- In order to map the Dipole into the Quadrupole, we use the polar forms of the functions;

$$w = |w|e^{i\phi} \quad \text{and} \quad z = |z|e^{i\theta}$$

Since $w = \dfrac{z^2}{h}$ was used to convert the quadrupole into the dipole,

$$z^2 = hw = h|w|e^{i\phi}$$

$$z = \sqrt{h|w|}e^{i\frac{\phi}{2}} = |z|e^{i\theta} \quad \text{therefore;} \quad |z| = \sqrt{h|w|} \quad \text{and} \quad \theta = \frac{\phi}{2}$$

Finally;

$$x = |z|\cos\theta = \sqrt{h|w|}\cos\frac{\phi}{2}$$

$$y = |z|\sin\theta = \sqrt{h|w|}\sin\frac{\phi}{2}$$

Mapping a Dipole into a Sextupole

- In order to map the Dipole into the Sextupole, we use the polar forms of the functions;

$$w = |w|e^{i\phi} \quad \text{and} \quad z = |z|e^{i\theta}$$

Since $w = \frac{z^3}{h^2}$ was used to convert the quadrupole into the dipole,

$$z^3 = h^2 w = h^2 |w| e^{i\phi}$$

$$z = \sqrt[3]{h^2|w|}\, e^{i\frac{\phi}{3}} = |z|e^{i\theta} \quad \text{therefore;} \quad |z| = \sqrt[3]{h^2|w|} \quad \text{and} \quad \theta = \frac{\phi}{3}$$

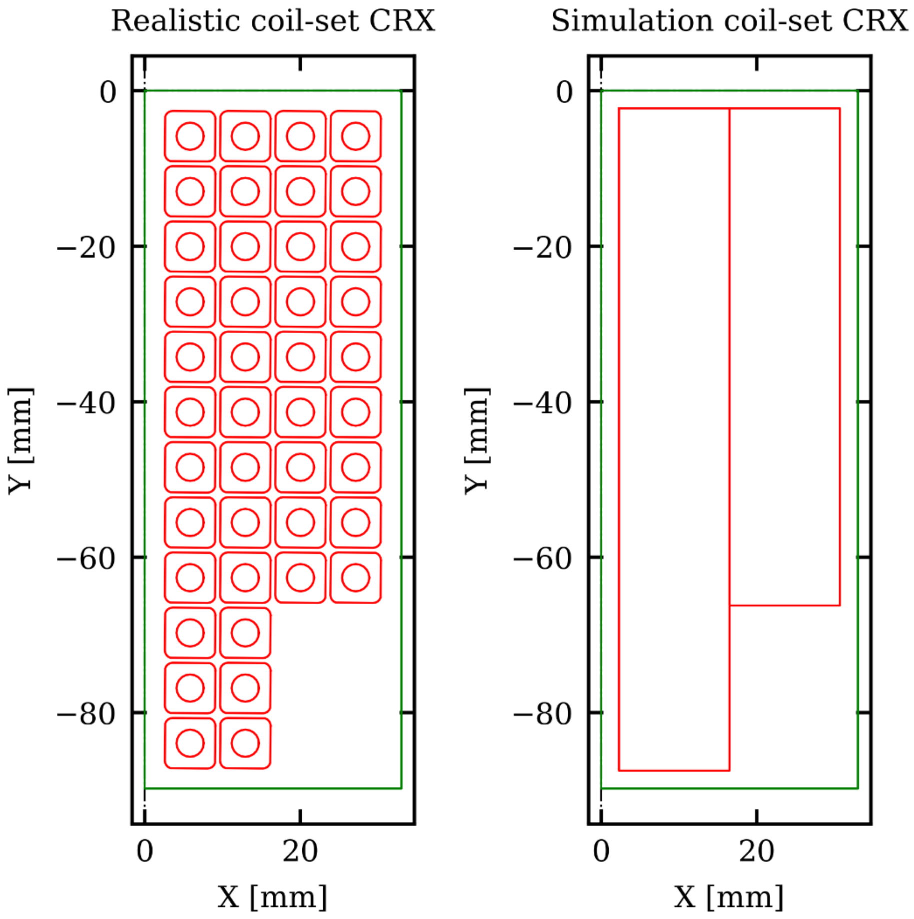
Finally;

$$x = |z|\cos\theta = \sqrt[3]{h^2|w|}\cos\frac{\phi}{3}$$

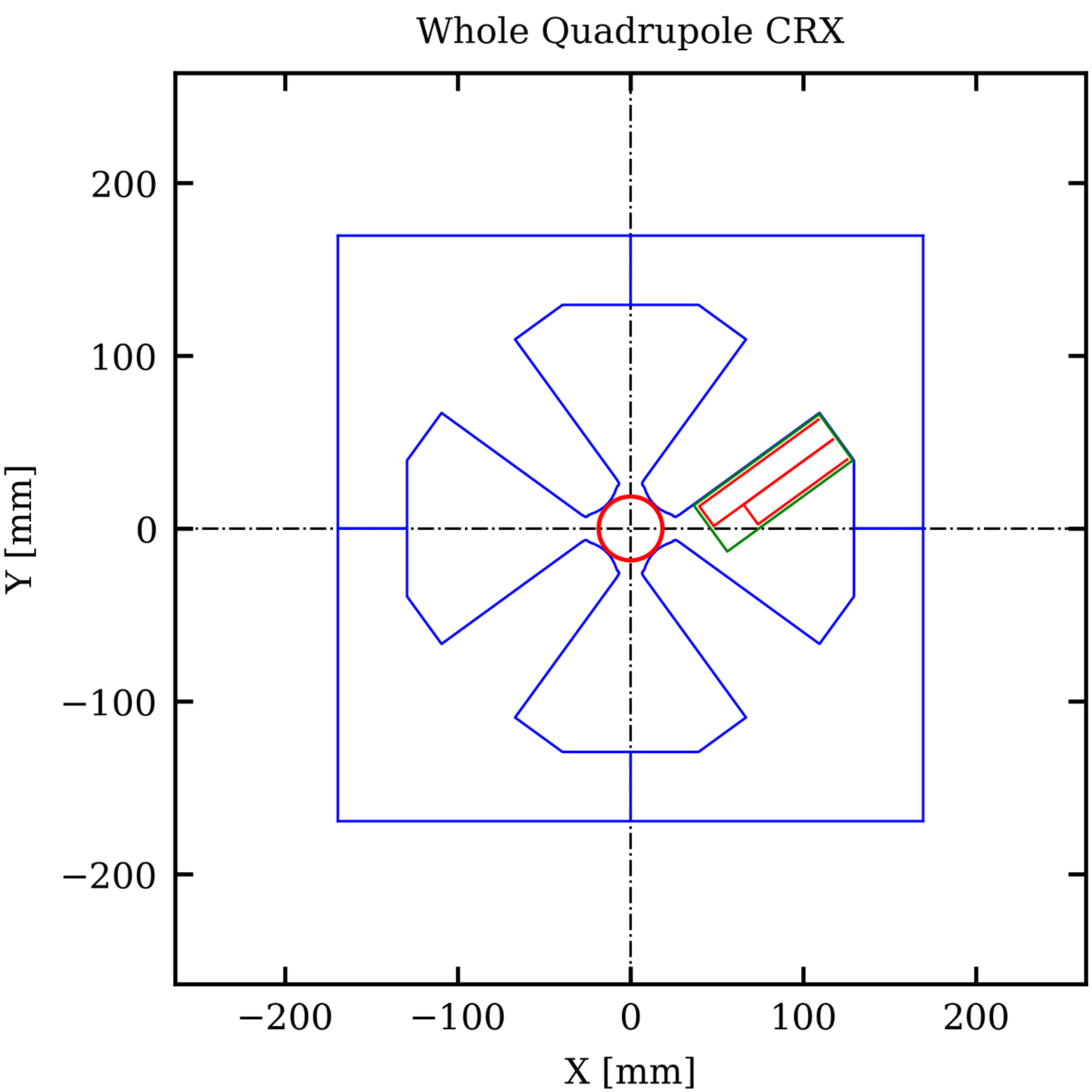
$$y = |z|\sin\theta = \sqrt[3]{h^2|w|}\sin\frac{\phi}{3}$$

Quadrupole

- Coil-set configuration



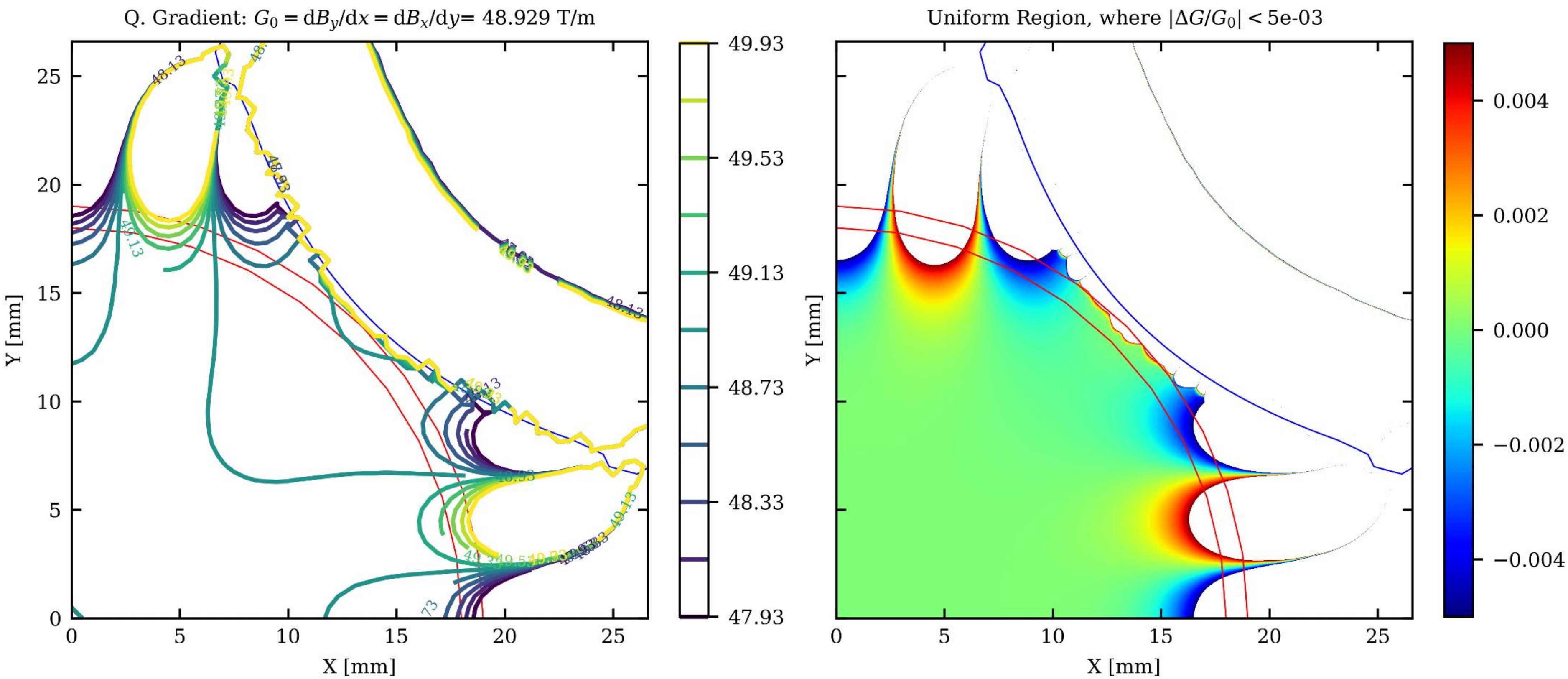
Quadrupole



Whole Quadrupole CRX
200
100
0
−100
−200
Y [mm]
−200
−100
0
100
200
X [mm]

Quadrupole

- 2D Field-map optimization



끝

Iron dominated

Coil dominated

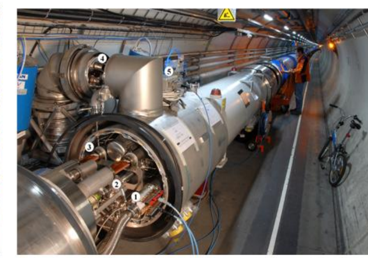
Normal conducting

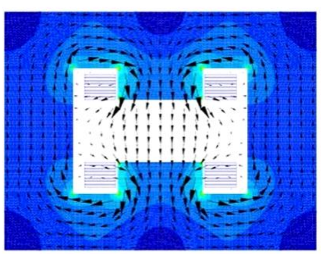
Superferric

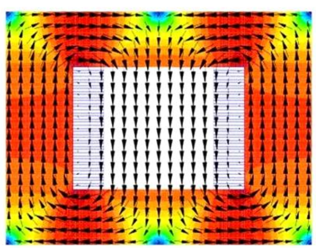
Permanent magnet

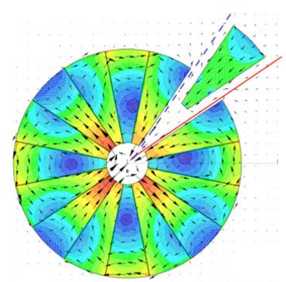
Class 1
large area,
“medium” field

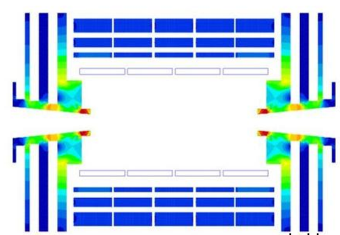
Class 2
Small area
high **B**, high **J**

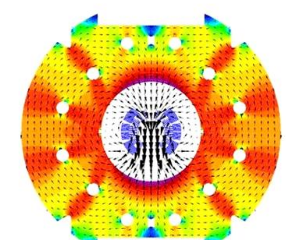






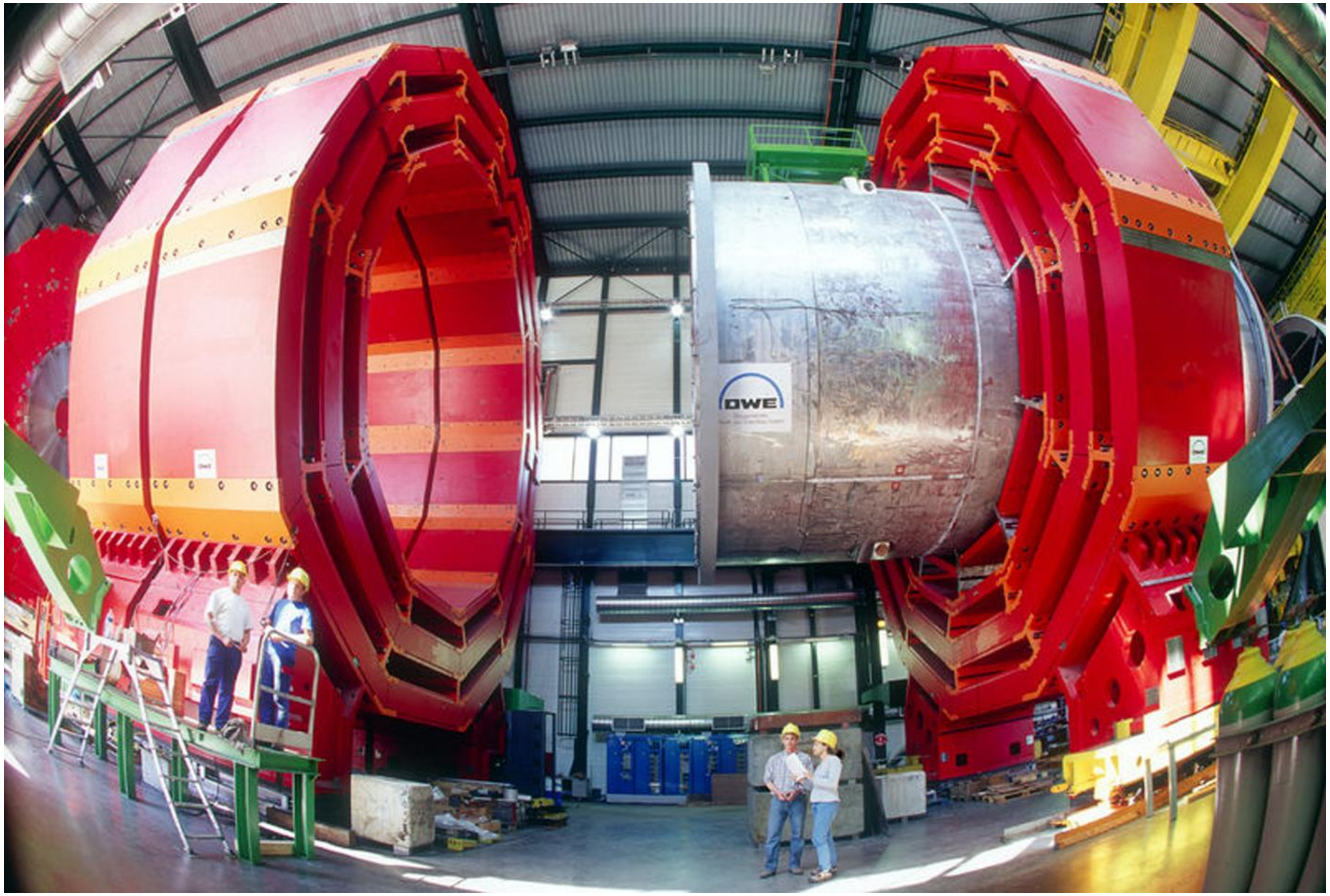






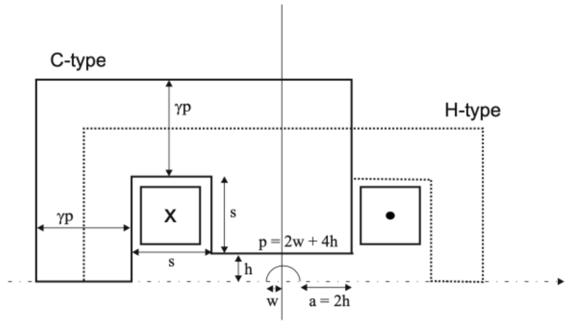
출처: CERN S. Russenchucks 강의노트

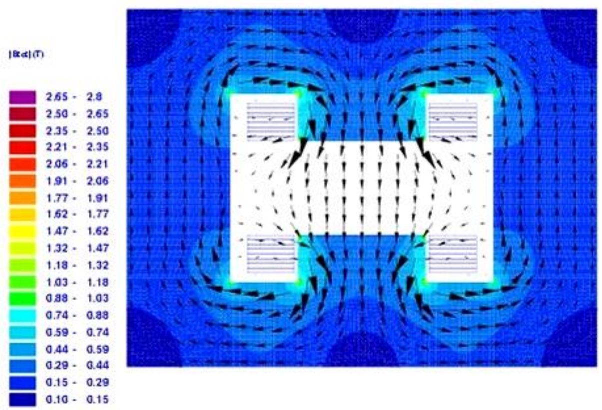
Detector Magnet



출처: CERN S. Russenchucks 강의노트

H Magnet (LHC Transfer Line)

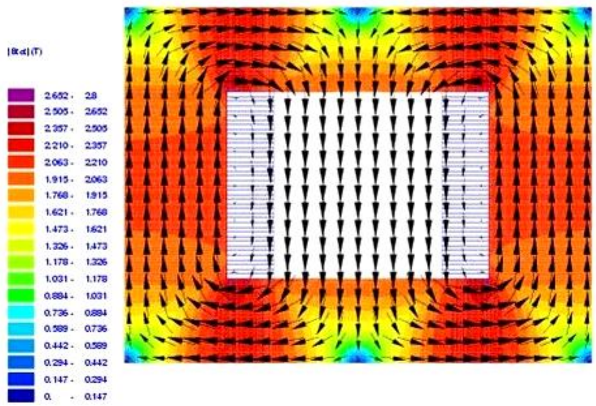




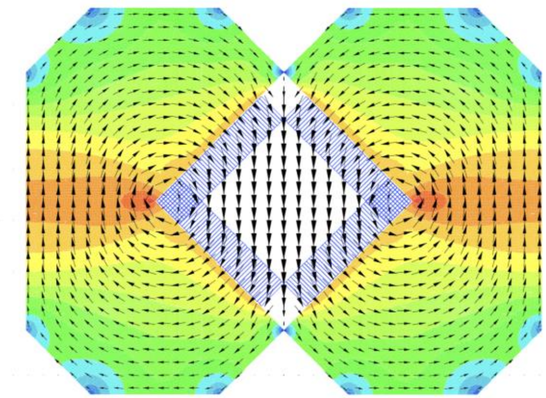
$N \cdot I = 24000$ A $\quad B_1 = 0.3$ T $\quad B_s = 0.065$ T $\quad$ Fill.fac. 0.98

출처: CERN S. Russenchucks 강의노트

Window Frame Magnet



$N \cdot I = 360$ kA, $B_t = 2.0$ T, $B_s = 1.04$ T



$N \cdot I = 625$ kA, $B_t = 2.38$ T, $B_s = 1.36$ T

출처: CERN S. Russenchucks 강의노트