특론: 가속기 실험실습 I

(NUCE719P-01/PHYS715P-01, 정모세)

eLABs 시설을 이용한 빔운전 및 RF/빔진단 기초 2
(부제: RF 기초 2 – UNIST 최은미 교수님 여름학교 강의 내용)

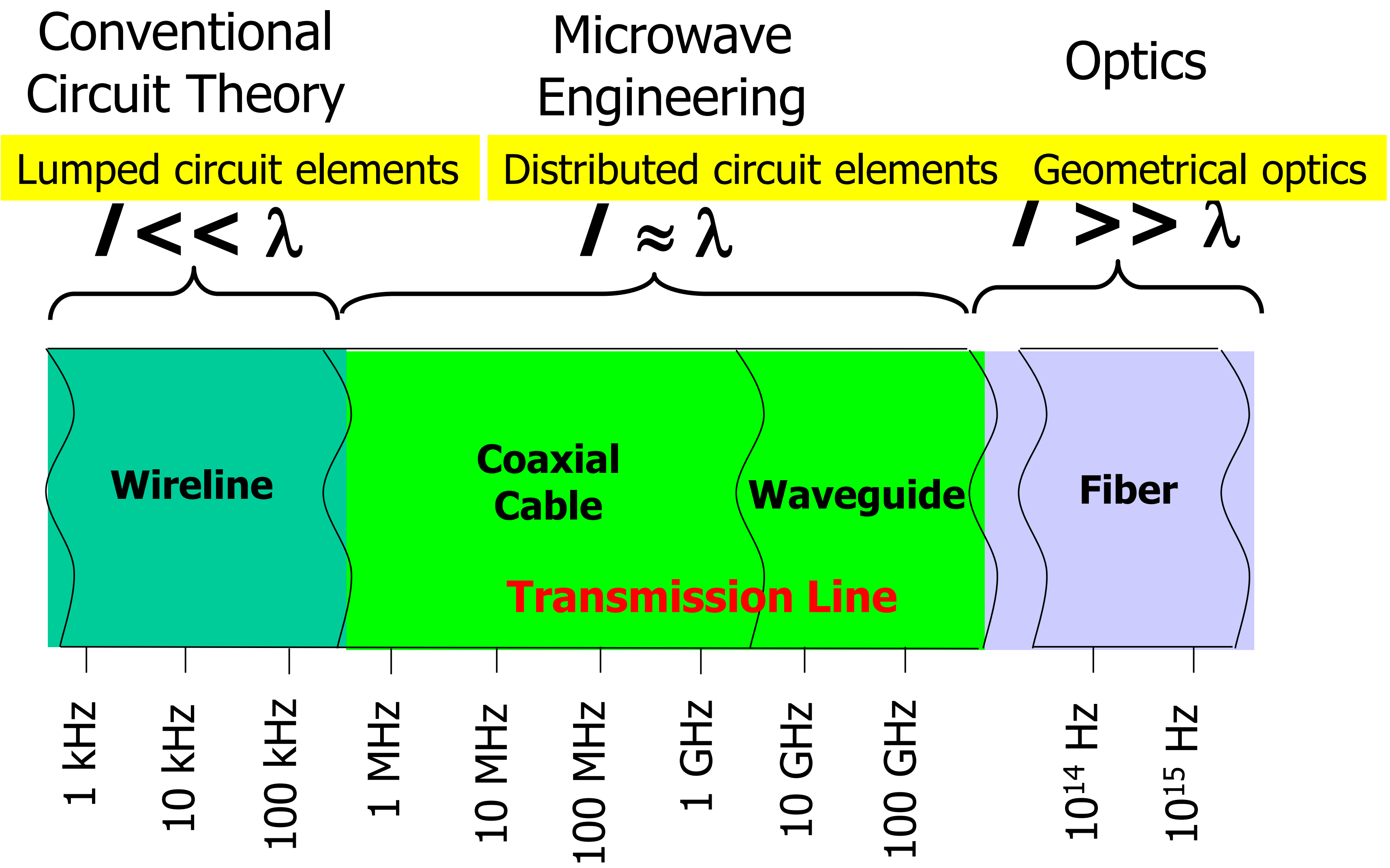
정모세

첨단원자력공학부 & 물리학과
moses@postech.ac.kr, 제1실험동 303호

Contents

Lumped circuit model in TL

- For high frequency, standard circuit theory is failed to solve microwave network problems.



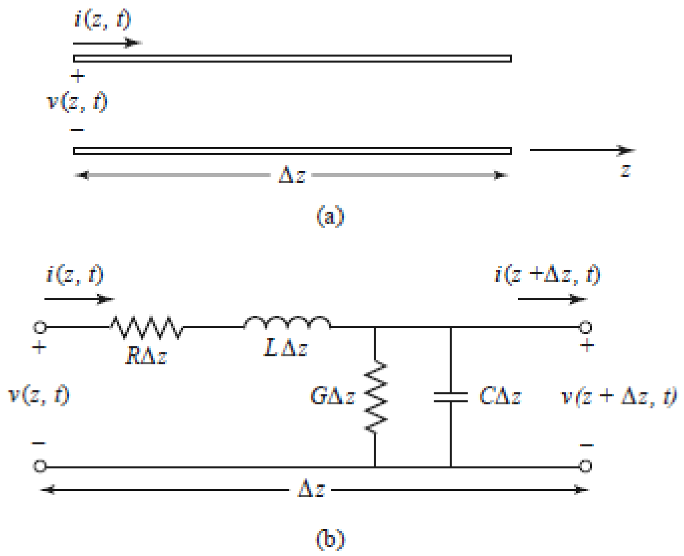
Advantages of microwaves

- Wide bandwidth (due to high frequency)
- Smaller size
- More available frequency spectrum
- Better resolution radar with high frequency
- High antenna gain

Disadvantages of microwaves

- Expensive components
- High signal losses
- Manufacturing difficulty

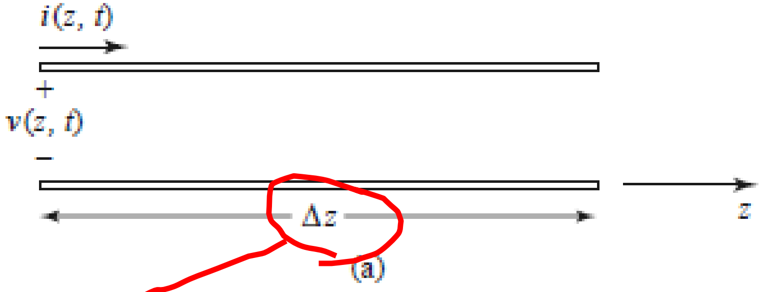
Lumped circuit model in TL



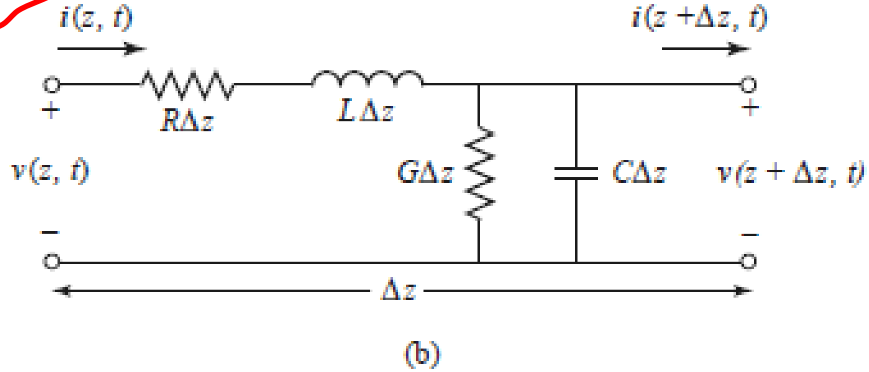
Circuit theory vs. **Transmission line theory**

Electrical size	$L \ll \lambda$	$L \sim \lambda$
	Lumped element: mag & phase of V,I do not vary	Distributed element: mag & phase of V,I vary

TL can be represented as a two-wire line (since TEM wave propagation always needs at least two conductors)



The piece of line can be modeled as a lumped circuit



R = series resistance per unit length, for both conductors, in Ω/m.
L = series inductance per unit length, for both conductors, in H/m.
G = shunt conductance per unit length, in S/m.
C = shunt capacitance per unit length, in F/m.

- Kirchhoff's voltage law applies:

$$v(z,t) - R\Delta z i(z,t) - L\Delta z \frac{\partial i(z,t)}{\partial t} - v(z+\Delta z,t) = 0,$$

- Kirchhoff's current law applies:

$$i(z,t) - G\Delta z v(z+\Delta z,t) - C\Delta z \frac{\partial v(z+\Delta z,t)}{\partial t} - i(z+\Delta z,t) = 0.$$

Telegrapher's equation

미래기반 가속기
전문인력양성 사업단

$$\frac{\partial v(z,t)}{\partial z} = -Ri(z,t) - L\frac{\partial i(z,t)}{\partial t},$$

$$\frac{\partial i(z,t)}{\partial z} = -Gv(z,t) - C\frac{\partial v(z,t)}{\partial t}.$$

$$v(z,t) = V(z)e^{j\omega t}$$

$$i(z,t) = I(z)e^{j\omega t}$$

$$\frac{dV(z)}{dz} = -(R + j\omega L)I(z),$$

$$\frac{dI(z)}{dz} = -(G + j\omega C)V(z).$$

$$\frac{d^2V(z)}{dz^2} - \gamma^2 V(z) = 0,$$

$$\frac{d^2I(z)}{dz^2} - \gamma^2 I(z) = 0,$$

$$V(z) = V_o^+ e^{-\gamma z} + V_o^- e^{\gamma z},$$

$$I(z) = I_o^+ e^{-\gamma z} + I_o^- e^{\gamma z},$$

where $\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)}$

Wave propagation in TL

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$$\boxed{\begin{aligned} V(z) &= V_o^+ e^{-\gamma z} + V_o^- e^{\gamma z}, \\ I(z) &= I_o^+ e^{-\gamma z} + I_o^- e^{\gamma z}, \end{aligned}}$$

$$\frac{dV(z)}{dz} = -(R + j\omega L)I(z)$$

$$\Downarrow$$

$$I(z) = \frac{\gamma}{R + j\omega L}\left(V_o^+ e^{-\gamma z} - V_o^- e^{\gamma z}\right) = \frac{1}{Z_0} V_0^+ e^{-\gamma z} - \frac{1}{Z_0} V_0^- e^{\gamma z}$$

$$\frac{V_o^+}{I_o^+} = Z_0 = \frac{-V_o^-}{I_o^-}.$$

- Characteristic impedance, Z_0

$$\boxed{Z_0 = \frac{R + j\omega L}{\gamma} = \sqrt{\frac{R + j\omega L}{G + j\omega C}},}$$

Converting back to time domain $\longrightarrow$

$$\begin{aligned} v(z, t) = {} & |V_o^+| \cos(\omega t - \beta z + \phi^+) e^{-\alpha z} \\ & + |V_o^-| \cos(\omega t + \beta z + \phi^-) e^{\alpha z} \end{aligned}$$

$$\lambda = \frac{2\pi}{\beta}$$

The lossless line

- In most cases, the loss of the line is very small, and can be neglected

Let $R = G = 0$

$$\gamma = \alpha + j\beta = \sqrt{(\cancel{R} + j\omega L)(\cancel{G} + j\omega C)} = j\omega\sqrt{LC}$$

$$Z_0 = \frac{R + j\omega L}{\gamma} = \sqrt{\frac{\cancel{R} + j\omega L}{\cancel{G} + j\omega C}} = \sqrt{\frac{L}{C}}$$ = REAL

Therefore

$$\boxed{\begin{aligned} V(z) &= V_o^+ e^{-j\beta z} + V_o^- e^{j\beta z}, \\ I(z) &= \frac{V_o^+}{Z_0} e^{-j\beta z} - \frac{V_o^-}{Z_0} e^{j\beta z}. \end{aligned}}$$

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{\omega\sqrt{LC}} \qquad v_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}}$$

(L 과 C 의 단위 조심)

Telegrapher equations derived from field analysis of a coaxial line

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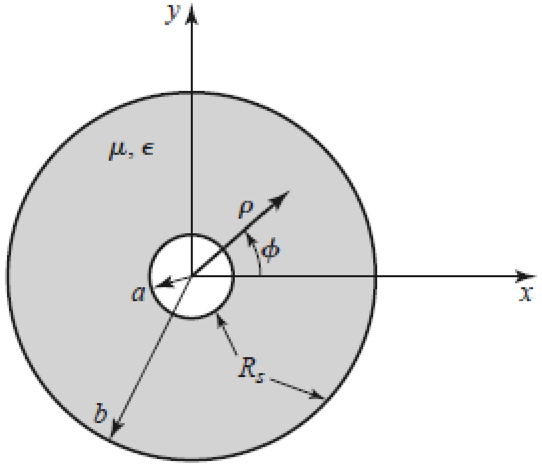
Telegrapher equation previously:

$$\frac{dV(z)}{dz} = -(R + j\omega L)I(z),$$

$$\frac{dI(z)}{dz} = -(G + j\omega C)V(z).$$

Consider coaxial line



Starting from Maxwell's curl equations

$$\nabla \times \bar{E} = -j\omega\mu\bar{H},$$

$$\nabla \times \bar{H} = j\omega\epsilon\bar{E},$$

$$\epsilon = \epsilon' - j\epsilon''$$

(dielectric loss allowed, conductor loss ignored)

$$-\hat{\rho}\frac{\partial E_\phi}{\partial z} + \hat{\phi}\frac{\partial E_\rho}{\partial z} + \hat{z}\frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho E_\phi) = -j\omega\mu(\hat{\rho}H_\rho + \hat{\phi}H_\phi),$$

$$-\hat{\rho}\frac{\partial H_\phi}{\partial z} + \hat{\phi}\frac{\partial H_\rho}{\partial z} + \hat{z}\frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho H_\phi) = j\omega\epsilon(\hat{\rho}E_\rho + \hat{\phi}E_\phi).$$

$E_z = H_z = 0$ (for TEM wave)

$\partial/\partial\phi = 0$ (due to azimuthal sym)

Telegrapher equations derived from field analysis of a coaxial line

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$$\therefore \quad E_\phi = \frac{f(z)}{\rho},$$

$$H_\phi = \frac{g(z)}{\rho}.$$

And boundary condition: $E_\phi = 0$ at $\rho = a,\ b$

$\boxed{E_\phi = 0}$ Everywhere to satisfy b'dary condition

then

$$-\hat{\rho}\underbrace{\frac{\partial E_\phi}{\partial z}}_{0} + \hat{\phi}\frac{\partial E_\rho}{\partial z} + \hat{z}\frac{1}{\rho}\underbrace{\frac{\partial}{\partial \rho}(\rho E_\phi)}_{0} = -j\omega\mu(\hat{\rho}H_\rho + \hat{\phi}H_\phi),$$

therefore $\boxed{H_\rho = 0}$

$$\therefore \quad \boxed{\begin{aligned} \frac{\partial E_\rho}{\partial z} &= -j\omega\mu H_\phi, \\ \frac{\partial H_\phi}{\partial z} &= -j\omega\epsilon E_\rho. \end{aligned}}$$

since $H_\phi = \dfrac{g(z)}{\rho}$

from $\dfrac{\partial H_\phi}{\partial z} = -j\omega\epsilon E_\rho. \longrightarrow E_\rho = \dfrac{h(z)}{\rho}$

Telegrapher equations derived from field analysis of a coaxial line

Therefore

$$\frac{\partial E_\rho}{\partial z} = -j\omega\mu H_\phi,$$

$$\frac{\partial H_\phi}{\partial z} = -j\omega\epsilon E_\rho.$$

$$\longrightarrow$$

$$\boxed{\begin{aligned}\frac{\partial h(z)}{\partial z} &= -j\omega\mu g(z)\\ \frac{\partial g(z)}{\partial z} &= -j\omega\epsilon h(z)\end{aligned}}$$

$$V(z) = \int_{\rho=a}^{b} E_\rho(\rho, z)d\rho = h(z)\int_{\rho=a}^{b}\frac{d\rho}{\rho} = h(z)\ln\frac{b}{a},$$

$$I(z) = \int_{\phi=0}^{2\pi} H_\phi(a, z)a\,d\phi = 2\pi g(z)$$

$$\therefore \quad \frac{\partial V(z)}{\partial z} = -j\frac{\omega\mu\ln b/a}{2\pi}I(z),$$

$$\frac{\partial I(z)}{\partial z} = -j\omega(\epsilon' - j\epsilon'')\frac{2\pi V(z)}{\ln b/a}.$$

Finally, Telegrapher eqn:

$$\boxed{\begin{aligned}\frac{\partial V(z)}{\partial z} &= -j\omega L I(z),\\ \frac{\partial I(z)}{\partial z} &= -(G + j\omega C)V(z)\end{aligned}}$$

The terminated lossless transmission line

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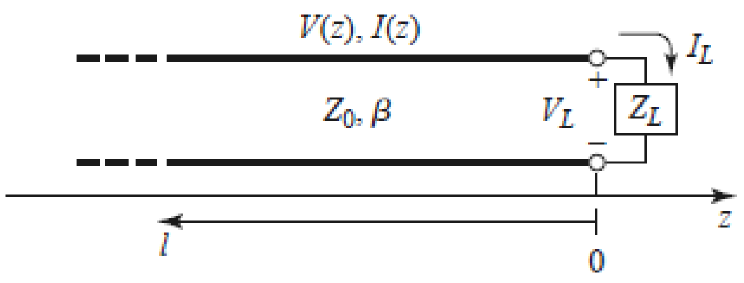
TL terminated in a load impedance, Z_L



Incident wave: $V_o^+ e^{-j\beta z}$ generated z<0 from source

The ratio V/I, characteristic impedance of the line, Z_0:

$Z_L \neq Z_0$ → reflected wave occurs

Total voltage: $$V(z) = V_o^+ e^{-j\beta z} + V_o^- e^{j\beta z}$$

Total current: $$I(z) = \frac{V_o^+}{Z_0} e^{-j\beta z} - \frac{V_o^-}{Z_0} e^{j\beta z}$$

At $z = 0$ $$Z_L = \frac{V(0)}{I(0)} = \frac{V_o^+ + V_o^-}{V_o^+ - V_o^-} Z_0$$

$$\therefore \quad \boxed{V_o^- = \frac{Z_L - Z_0}{Z_L + Z_0} V_o^+}$$

The terminated lossless transmission line

- Voltage reflection coefficient, Γ $\boxed{\Gamma = \dfrac{V_o^-}{V_o^+} = \dfrac{Z_L - Z_0}{Z_L + Z_0}}$ defined at the load

$$\therefore \quad V(z) = V_o^+\left(e^{-j\beta z} + \Gamma e^{j\beta z}\right),$$
$$I(z) = \frac{V_o^+}{Z_0}\left(e^{-j\beta z} - \Gamma e^{j\beta z}\right).$$

→ Voltage and current are superposition of incident and reflected waves.
→ Only for Γ=0, no reflected waves. In this case, Z_L=Z_0. "**matched**"

- Time-averaged power flow

$$P_{\text{avg}} = \frac{1}{2}\text{Re}\{V(z)I(z)^*\} = \frac{1}{2}\frac{|V_o^+|^2}{Z_0}\text{Re}\{1 - \underbrace{\Gamma^* e^{-2j\beta z} + \Gamma e^{2j\beta z}} - |\Gamma|^2\}$$

pure imaginary (A-A* form)

$$= \frac{1}{2}\frac{|V_o^+|^2}{Z_0}\left(1 - |\Gamma|^2\right)$$

→ Average power flow is constant at any z.
→ Total power delivered to the load = incident power – reflected power
→ If, Γ=0: maximum power delivered to the load.
If |Γ|=1, no power delivered.

The terminated lossless transmission line

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- Return loss (RL)

Since not all power from generator is delivered to the load. This loss,

$$\mathrm{RL} = -20\log|\Gamma|\ \mathrm{dB}$$

→ for $\Gamma = 0$ → RL → ∞. For $|\Gamma| = 1$ → $RL = 0$ [dB]
→ $RL > 0$ for passive network (since $0 \le |\Gamma| \le 1$)

- If the load is matched to the line (Γ=0)

$|V(z)| = |V_o^+|$ → This line is said to be "flat"

- If the load is mismatched → reflected wave results in standing waves

$$\begin{aligned} V(z) = V_o^+\left(e^{-j\beta z} + \Gamma e^{j\beta z}\right) &= |V_o^+||1 + \Gamma e^{2j\beta z}| \\ &= |V_o^+||1 + \Gamma e^{-2j\beta\ell}| \\ &= |V_o^+||1 + |\Gamma|e^{j(\theta-2\beta\ell)}| \end{aligned} \qquad \Gamma = |\Gamma|e^{j\theta}$$

- Vmax occurs when $e^{j(\theta-2\beta\ell)} = 1$ $\quad V_{\max} = |V_o^+|(1 + |\Gamma|)$

- Vmin occurs when $e^{j(\theta-2\beta\ell)} = -1$ $\quad V_{\min} = |V_o^+|(1 - |\Gamma|)$

The terminated lossless transmission line

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- The standing wave ratio (SWR, VSWR): the ratio of V_{max} to V_{min}
  a measure of the mismatch of a line

$$\boxed{\mathrm{SWR} = \frac{V_{\max}}{V_{\min}} = \frac{1+|\Gamma|}{1-|\Gamma|}} \qquad 1 \leq \mathrm{SWR} \leq \infty$$

SWR = 1 for a matched load

- The distance between two successive Vmax (or Vmin)

$$\ell = 2\pi/2\beta = \pi\lambda/2\pi = \lambda/2$$

- The distance between Vmax and Vmin

$$\ell = \pi/2\beta = \lambda/4$$

- The reflection coeff can be generalized at any point *l*

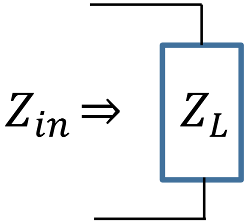
$$\Gamma = \frac{V_o^-}{V_o^+} = \frac{Z_L - Z_0}{Z_L + Z_0} \quad \text{at } l\text{=0} \qquad \boxed{\Gamma(\ell) = \frac{V_o^- e^{-j\beta\ell}}{V_o^+ e^{j\beta\ell}} = \Gamma(0)e^{-2j\beta\ell}} \quad \text{at } z = -l$$

$$V(z) = V_o^+ e^{-j\beta z} + V_o^- e^{j\beta z}$$

The terminated lossless transmission line

- As we have seen earlier, the real power flow is constant over z. But, Voltage for mismatched line, is oscillatory with z, therefore, one can expect the impedance seen looking into line is varying as z.

- Impedance seen looking into the load (Zin)



$$Z_{\text{in}} = \frac{V(-\ell)}{I(-\ell)} = \frac{V_o^+\left(e^{j\beta\ell} + \Gamma e^{-j\beta\ell}\right)}{V_o^+\left(e^{j\beta\ell} - \Gamma e^{-j\beta\ell}\right)} Z_0 = \frac{1 + \Gamma e^{-2j\beta\ell}}{1 - \Gamma e^{-2j\beta\ell}} Z_0$$

Since $\Gamma = \frac{V_o^-}{V_o^+} = \frac{Z_L - Z_0}{Z_L + Z_0}$

$$Z_{\text{in}} = Z_0 \frac{(Z_L + Z_0)e^{j\beta\ell} + (Z_L - Z_0)e^{-j\beta\ell}}{(Z_L + Z_0)e^{j\beta\ell} - (Z_L - Z_0)e^{-j\beta\ell}}$$

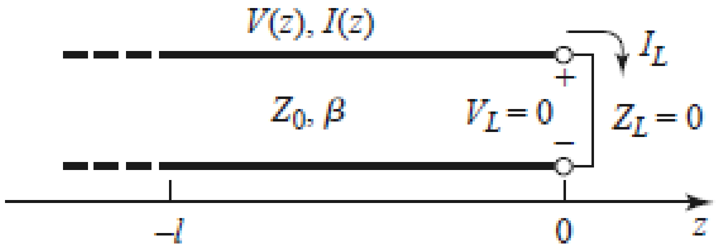
$$= Z_0 \frac{Z_L \cos\beta\ell + jZ_0 \sin\beta\ell}{Z_0 \cos\beta\ell + jZ_L \sin\beta\ell}$$

$$\boxed{= Z_0 \frac{Z_L + jZ_0 \tan\beta\ell}{Z_0 + jZ_L \tan\beta\ell}}$$

"TL impedance equation": input impedance of a length of TL with an arbitrary load impedance

A special case of lossless terminated lines

- TL terminated in a short circuit, $Z_L = 0$



$$\Gamma = \frac{V_o^-}{V_o^+} = \frac{Z_L - Z_0}{Z_L + Z_0} = \text{-1} \qquad SWR = \infty$$

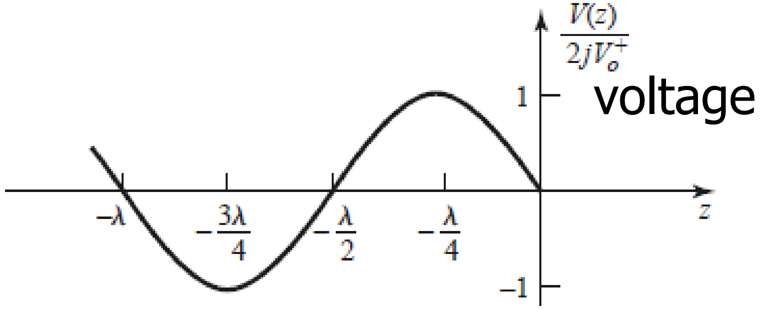
$$V(z) = V_o^+\left(e^{-j\beta z} - e^{j\beta z}\right) = -2j\,V_o^+ \sin\beta z,$$

$$I(z) = \frac{V_o^+}{Z_0}\left(e^{-j\beta z} + e^{j\beta z}\right) = \frac{2V_o^+}{Z_0}\cos\beta z,$$

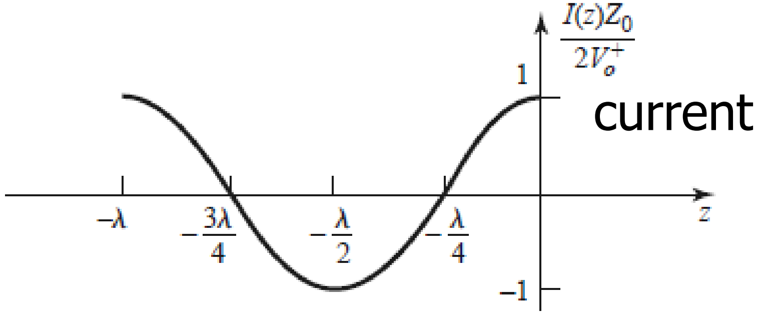
Here, at $z = 0, V = 0$, and $I = Imax$.

$Z_{in} = j\,Z_0 \tan\beta\ell$ purely imaginary (reactance for any length of l

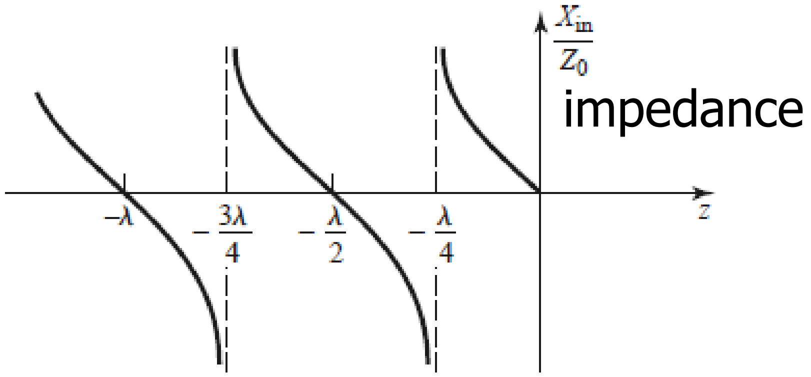
$$\begin{cases} Z_{in} = 0 & \text{for } l = 0 \\ Z_{in} = \infty & \text{for } l = \lambda/4 \end{cases}$$



(a)

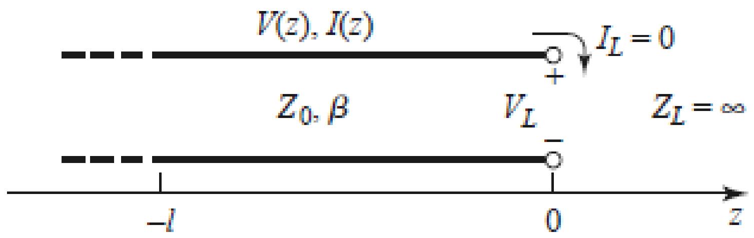


(b)



A special case of lossless terminated lines

- TL terminated in an open circuit, $Z_L = \infty$



$$\Gamma = \frac{V_o^-}{V_o^+} = \frac{Z_L - Z_0}{Z_L + Z_0} = 1 \qquad SWR = \infty$$

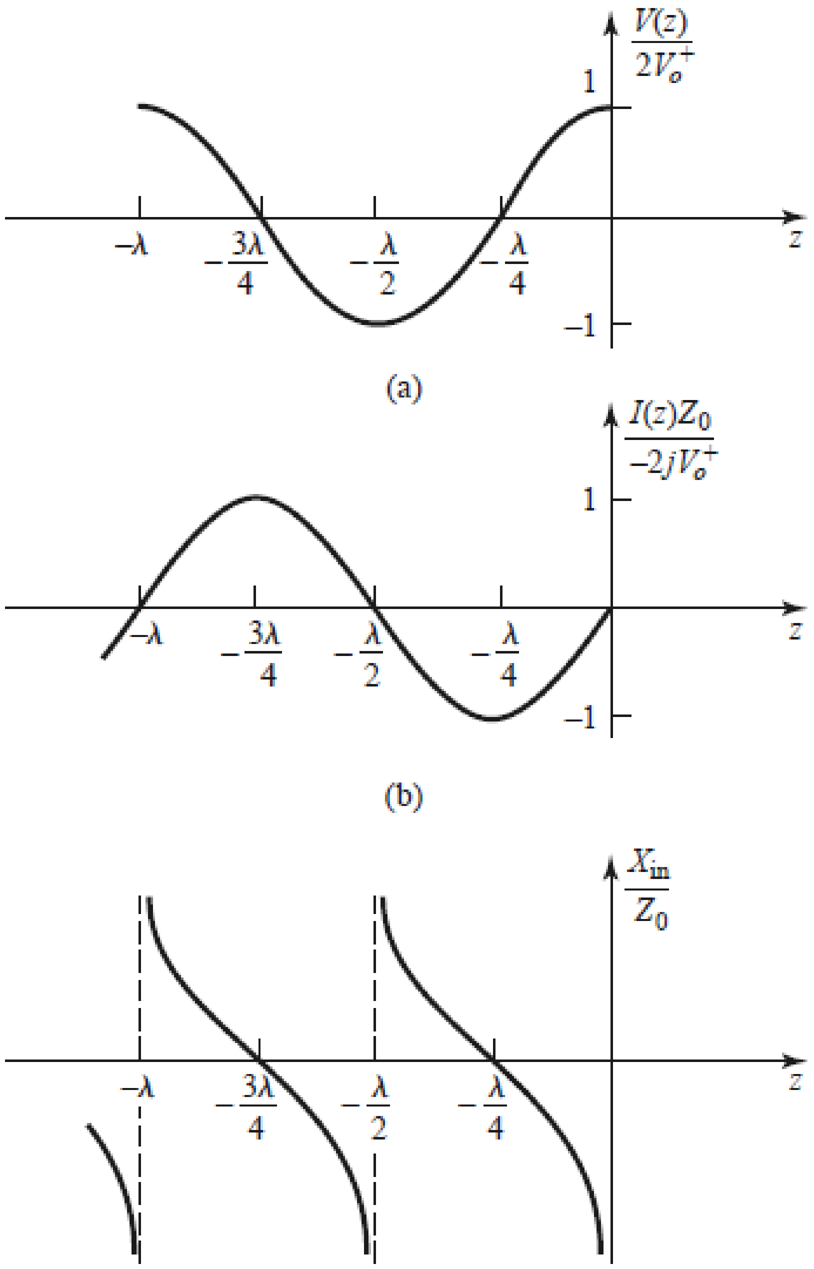
$$V(z) = V_o^+\left(e^{-j\beta z} + e^{j\beta z}\right) = 2V_o^+ \cos\beta z,$$

$$I(z) = \frac{V_o^+}{Z_0}\left(e^{-j\beta z} - e^{j\beta z}\right) = \frac{-2jV_o^+}{Z_0}\sin\beta z.$$

Here, at z=0, I=0, and V=Vmax.

$$Z_{in} = -jZ_0 \cot\beta\ell$$

purely imaginary (reactance for any length of *l*)



A special case of lossless terminated lines

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- TL terminated with special lengths: $l = \lambda/2$

$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan\beta\ell}{Z_0 + jZ_L \tan\beta\ell}$$
$$= Z_L$$

→ Half-wavelength line does not alter/transform the load impedance, regardless Z_0.

- TL terminated with special lengths: $l = \lambda/4 + n\lambda/2 \ (n = 1,2, \dots)$

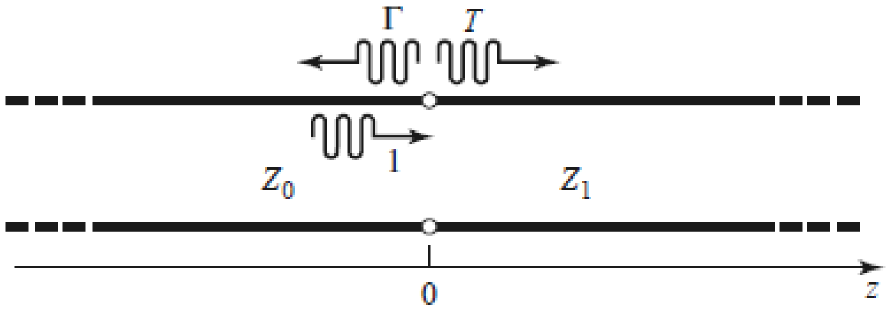
$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan\beta\ell}{Z_0 + jZ_L \tan\beta\ell}$$

$$= \frac{Z_0^2}{Z_L} \quad \text{or} \quad \frac{Z_{in}}{Z_0} = \frac{Z_0}{Z_L}$$

→ This kind of line is known to be "quarter-wave transformer" or, "λ/4 impedance transformer" (because it has the effect of transforming the load impedance in an inverse manner, depending on the Z_0 of the line.)

A special case of lossless terminated lines

- TL of characteristic impedance Z_0 feeding a line of different characteristic line, Z_1



- Assume that the load line is infinitely long, or terminated to its own characteristic impedance (no reflection at the far end)

- Input impedance seen by the feed line is Z_1

$$\Gamma = \frac{Z_1 - Z_0}{Z_1 + Z_0}$$

- Now, some incident wave reflected, and transmitted

$$\begin{cases} V(z) = V_o^+\left(e^{-j\beta z} + \Gamma e^{j\beta z}\right), & z < 0 \\ V(z) = V_o^+ T e^{-j\beta z} & \text{for } z > 0. \end{cases}$$

at z=0

$$T = 1 + \Gamma = 1 + \frac{Z_1 - Z_0}{Z_1 + Z_0} = \frac{2Z_1}{Z_1 + Z_0}$$

- Insertion loss (IL)

$$\boxed{\mathrm{IL} = -20 \log |T| \mathrm{dB}}$$

A special case of lossless terminated lines

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- Decibels and Nepers

The ratio of two power levels in dB

$$\boxed{10 \log \frac{P_1}{P_2} \text{ dB}}$$

$$P_1 = V_1^2/R_1 \text{ and } P_2 = V_2^2/R_2$$

$$\boxed{10 \log \frac{V_1^2 R_2}{V_2^2 R_1} = 20 \log \frac{V_1}{V_2}\sqrt{\frac{R_2}{R_1}} \text{ dB}}$$

$$\boxed{20 \log \frac{V_1}{V_2} \text{ dB}}$$

for the load resistances are equal

The ratio of two voltage levels in Np

$$\boxed{\ln \frac{V_1}{V_2} \text{ Np}}$$

$$\boxed{\frac{1}{2} \ln \frac{P_1}{P_2} \text{ Np}}$$

$$\frac{V_2}{V_1} = e$$

- A change in power by half = 3 dB
- A change in power by factor of 10 = 10 dB

Since 1 Np = a power ratio of e^2

$$1 \text{ Np} = 10 \log e^2 = 8.686 \text{ dB}$$

Absolute power in dB

$$\boxed{10 \log \frac{P_1}{1 \text{ mW}} \text{ dBm}}$$

Ex) $1mW = 0\ dBm$
$10mW = 10\ dBm$
$100mW = 20\ dBm$
$1000mW = 30\ dBm$

The Smith chart

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- Graphical aid for solving TL problems
- Polar plot of the voltage reflection coefficient, Γ

$$\Gamma = \frac{V_o^-}{V_o^+} = \frac{Z_L - Z_0}{Z_L + Z_0} \boxed{= |\Gamma| e^{j\theta}} \qquad |\Gamma| \leq 1 \qquad z = Z/Z_0$$

→ Convert Γ to <u>normalized</u> impedance (or admittance)

$$\boxed{\Gamma = \frac{z_L - 1}{z_L + 1} = |\Gamma| e^{j\theta}}$$

$z_L = Z_L / Z_0$: normalized load impedance

Z_0: characteristic impedance

or

$$\boxed{z_L = \frac{1 + |\Gamma| e^{j\theta}}{1 - |\Gamma| e^{j\theta}}}$$

Let

$$\begin{cases} \Gamma = \Gamma_r + j\Gamma_i \\ z_L = r_L + jx_L \end{cases}$$

$$r_L + jx_L = \frac{(1 + \Gamma_r) + j\Gamma_i}{(1 - \Gamma_r) - j\Gamma_i}$$

The Smith chart

$$r_L + jx_L = \frac{(1+\Gamma_r) + j\Gamma_i}{(1-\Gamma_r) - j\Gamma_i}$$

$$r_L = \frac{1-\Gamma_r^2-\Gamma_i^2}{(1-\Gamma_r)^2+\Gamma_i^2},$$

$$x_L = \frac{2\Gamma_i}{(1-\Gamma_r)^2+\Gamma_i^2}.$$

rearrange

$$\left(\Gamma_r - \frac{r_L}{1+r_L}\right)^2 + \Gamma_i^2 = \left(\frac{1}{1+r_L}\right)^2$$ Resistance circle

$$(\Gamma_r - 1)^2 + \left(\Gamma_i - \frac{1}{x_L}\right)^2 = \left(\frac{1}{x_L}\right)^2,$$ Reactance circle

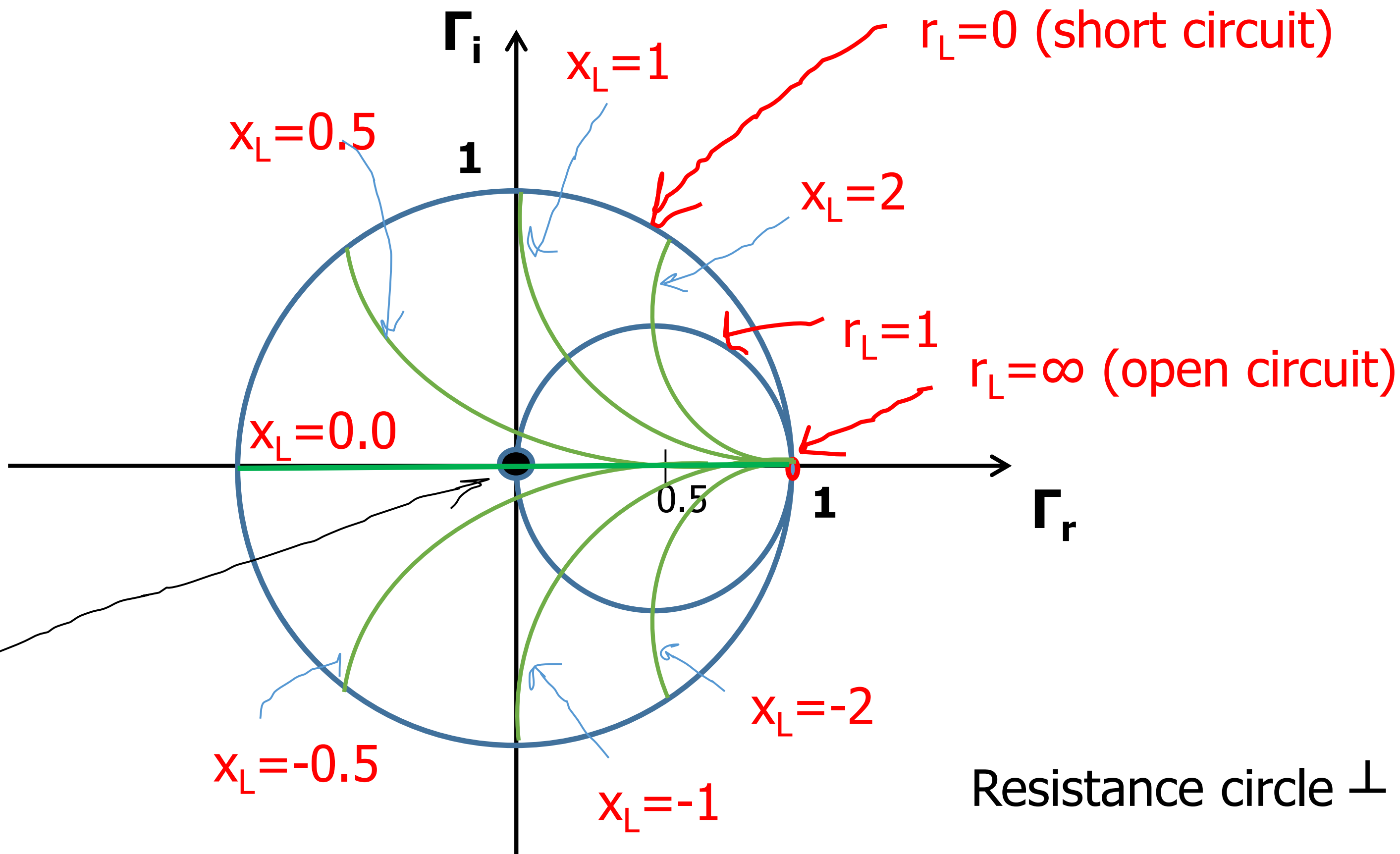
in Γ_r and Γ_i plane

Smith chart is very smart plot enabling complex values in one circle!

The Smith chart

$$\left(\Gamma_r - \frac{r_L}{1+r_L}\right)^2 + \Gamma_i^2 = \left(\frac{1}{1+r_L}\right)^2,$$ (Resistance circle)

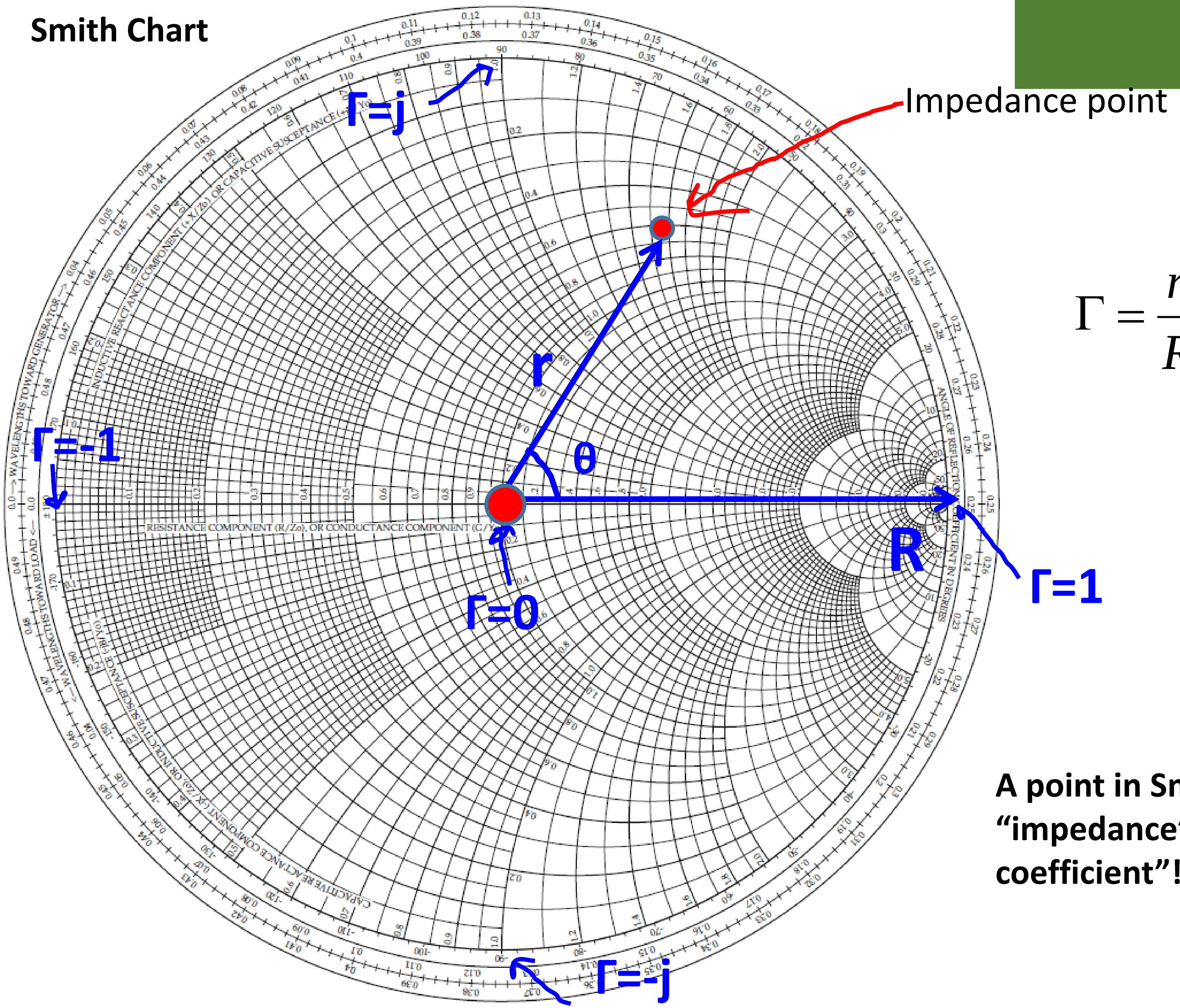
$$(\Gamma_r - 1)^2 + \left(\Gamma_i - \frac{1}{x_L}\right)^2 = \left(\frac{1}{x_L}\right)^2,$$ (Reactance circle)



Center: $z = 1 + j0 \rightarrow \Gamma = 0$
corresponding to
characteristic impedance

Resistance circle ⊥ Reactance circle

Smith Chart



$$\Gamma = \frac{r}{R} \angle \theta$$

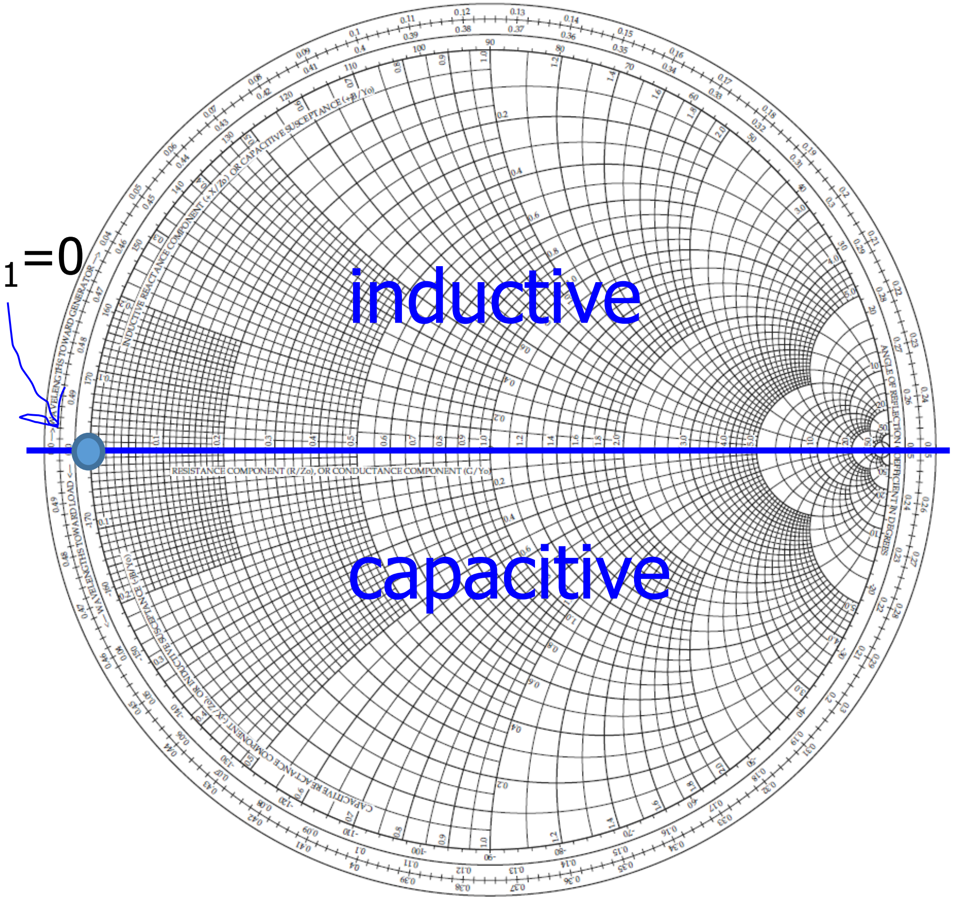
A point in Smith chart represent “impedance” as well as “reflection coefficient”!!!

The Smith chart

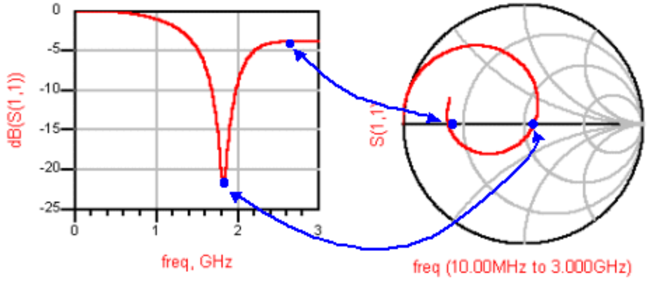
Often time, we know Γ first, and re-interpret Z, because we usually know S-parameter

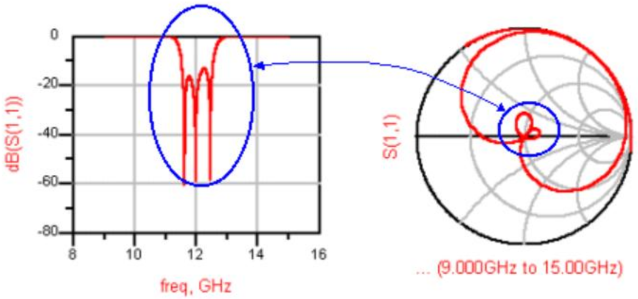
$$Z_L = R_L + jX = R_L + j\left(\omega L - \frac{1}{\omega C}\right)$$

S_{11}=0



- If S_{11} is small (reflection decrease) → plot approaches to the center
- If S_{11} is large (reflection increases) → plot approaches to the outer perimeter



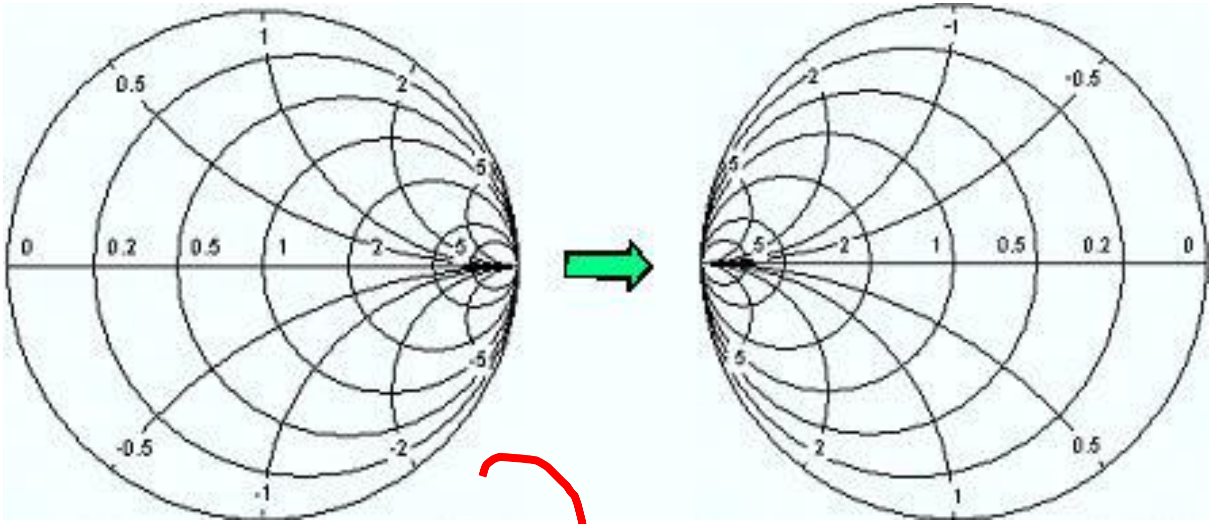


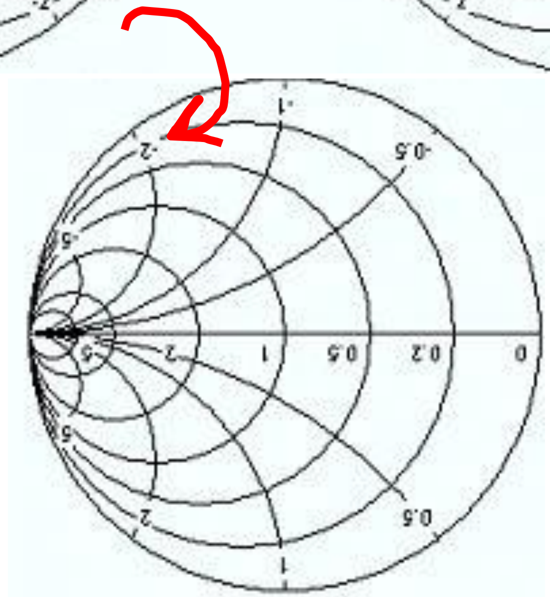
The Smith chart

- Admittance $y_L = \dfrac{1}{z_L}$

-impedance: how the signal is impeded
-admittance: how the signal is well passing

impedance admittance





Rotate impedance chart by 180deg

The Smith chart

Example: Basic Smith chart operations

A load impedance of 40 + *j* 70 Ω terminates 100 Ω transmission line what is 0.3 λ long. Find 1) the reflection coefficient at the load, 2) the reflection coefficient at the input to the line, 3) the input impedance, 4) standing wave ratio on the line, and 5) return loss.

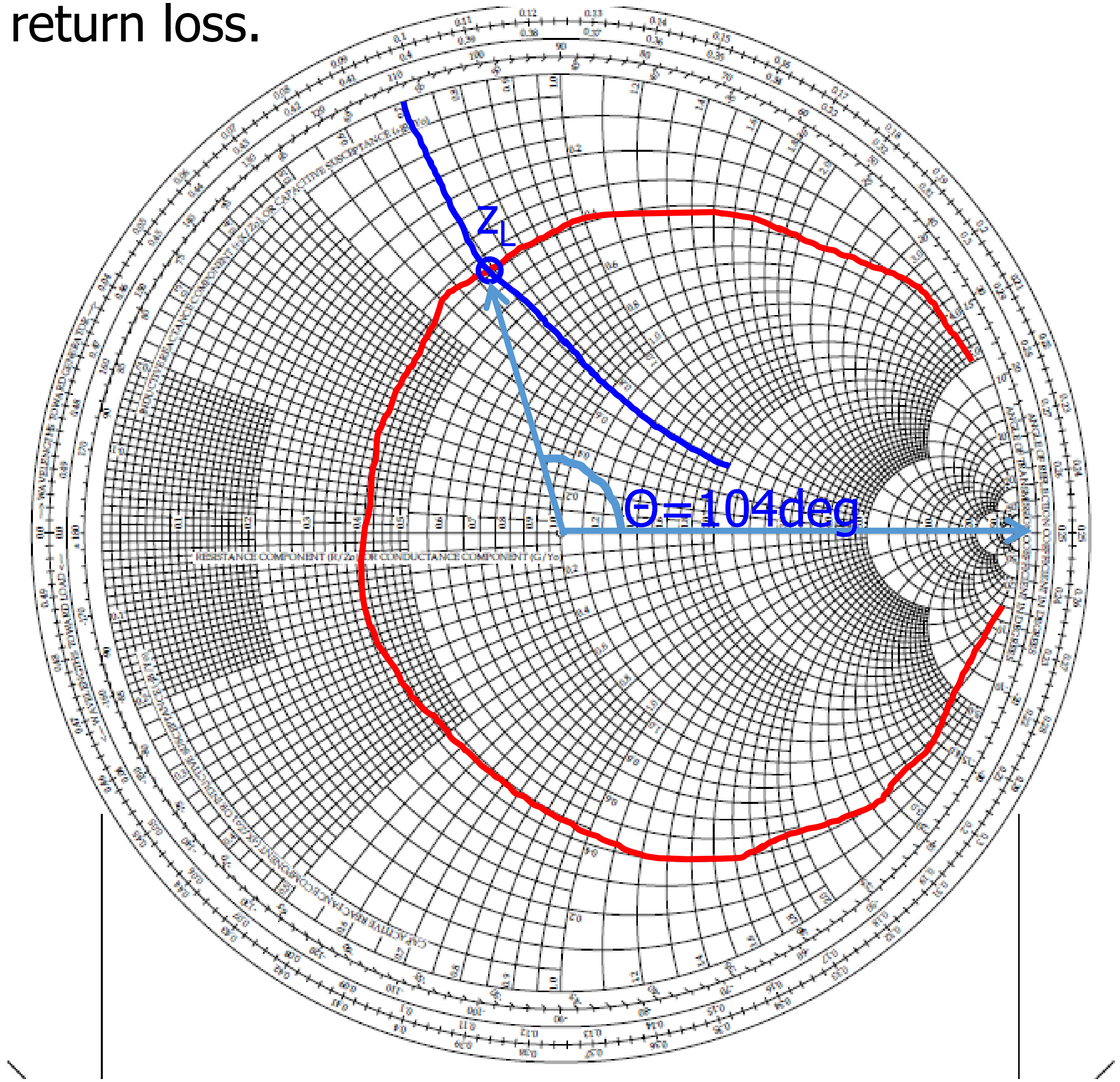
$$z_L = \frac{Z_L}{Z_0} = 0.4 + j0.7$$

1) Reflection coeff. @ the load:
By reading the length ratio

$$|\Gamma| = 0.59$$

4) $SWR = \dfrac{1+|\Gamma|}{1-|\Gamma|} = \dfrac{1+0.59}{1-0.59} = 3.87$

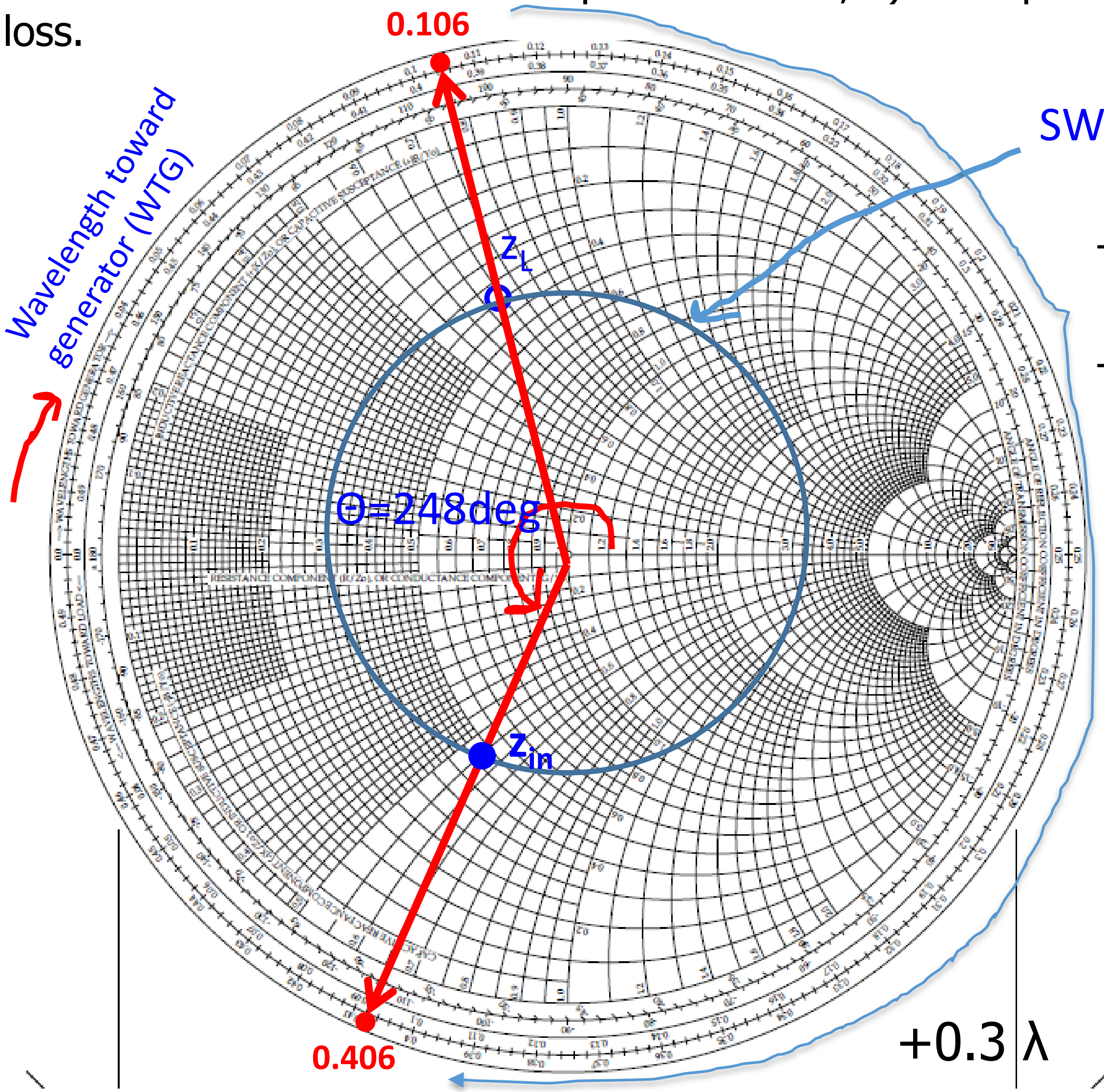
5) $RL = -20\log|\Gamma| = 4.6dB$



The Smith chart

Example: Basic Smith chart operations

A load impedance of 40 + j70 Ω terminates 100 Ω transmission line what is 0.3 λ long. Find 1) the reflection coefficient at the load, 2) the reflection coefficient at the input to the line, 3) the input impedance, 4) standing wave ratio on the line, and 5) return loss.



- WTG scale, the reference point reading is 0.106λ
- Bring the 0.106λ point by 0.3λ toward generator brings 0.406λ

3) therefore,
z_{in} = 0.365 – j 0.611

$$Z_{in} = Z_0 z_{in} = 100(0.635 - j0.611)$$
$$= 36.5 - j61.1\Omega$$

2) Still $|\Gamma| = 0.59$
but, phase Θ=248deg

More on the Smith chart

https://www.youtube.com/watch?v=iwpmX9oAhfg

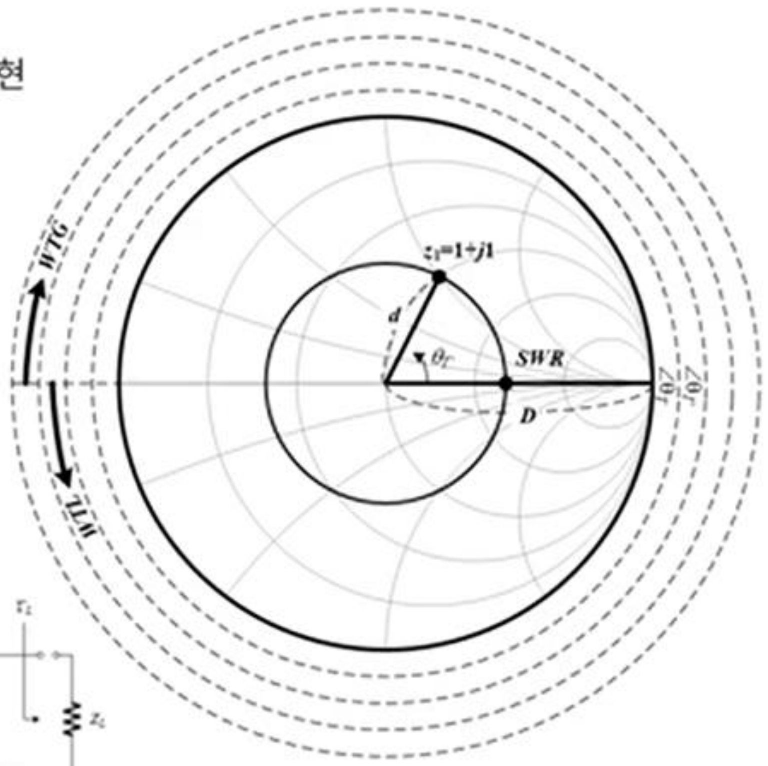
3.3 반사계수와 기타 파라미터

RFqna

전송선로 길이

스미스 차트 최외곽 원 외부 데이터 활용

- 영역 1과 영역 2는 투과파의 위상(θ_T)과 반사파의 위상(θ) 표현
  - 투과각은 전송선로 길이와 동일 → 한 바퀴는 180 deg
  - 반사각은 전송선로 길이의 2배 → 한 바퀴는 360 deg
- 영역 3과 영역 4는 파장 표현
  - WTG (Wavelength Toward Generator)
    : 소스 방향으로 증가하는 파장 표현
  - WTL (Wavelength Toward Load)
    : 부하 방향으로 증가하는 파장 표현
  - 일반적으로 부하에서 소스 방향으로 선로가 추가되면서 문제를 풀어나가기 때문에 WTL을 주로 이용함



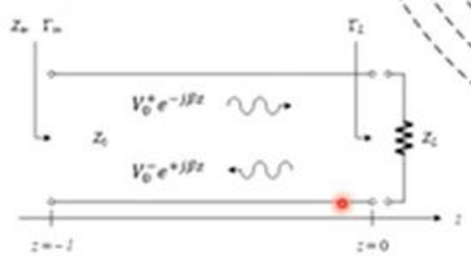
스미스 차트는 반사계수 표현 중심

- 스미스 차트의 한 바퀴는 반 파장 선로
  - 반사에 대한 파장과 위상은 선로 길이의 2배

$$\Gamma_{in} = \Gamma_L e^{-j2\beta l}$$

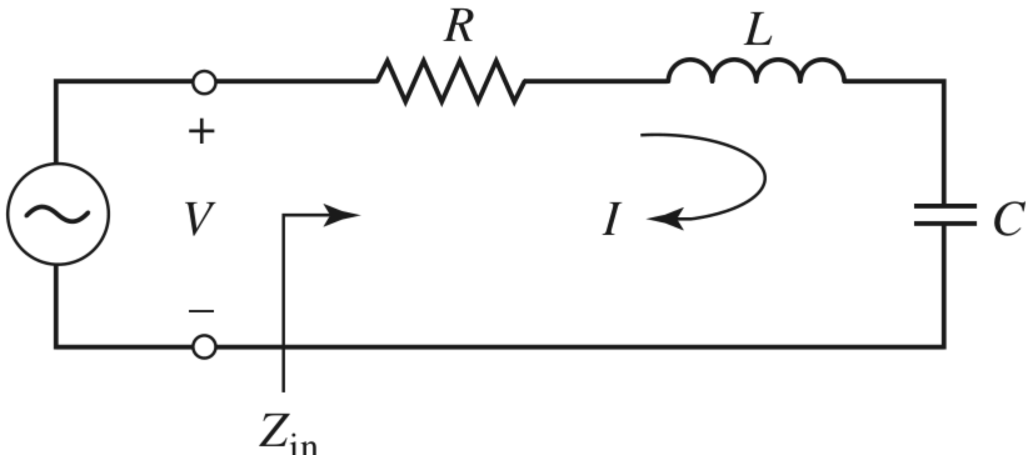
복소 평면에서의 회전

- 선로 길이가 반파장의 배수일 때 입력 임피던스는 동일함

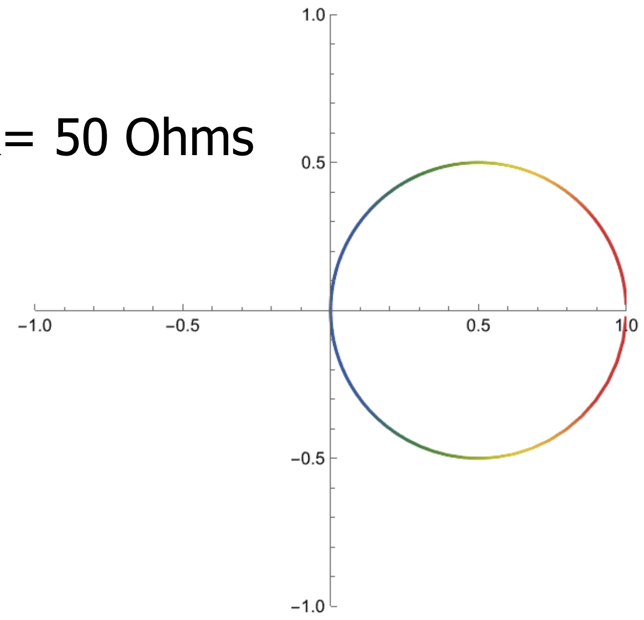


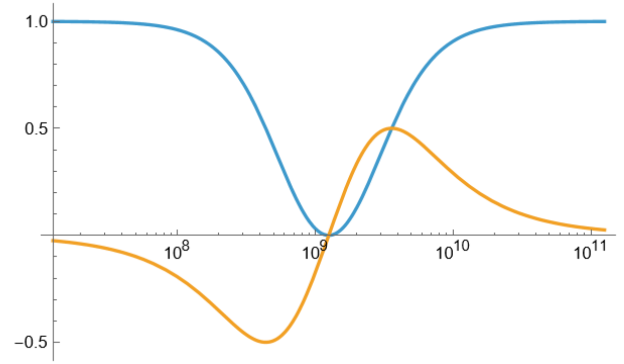
12

Series Resonant Circuit on the Smith chart

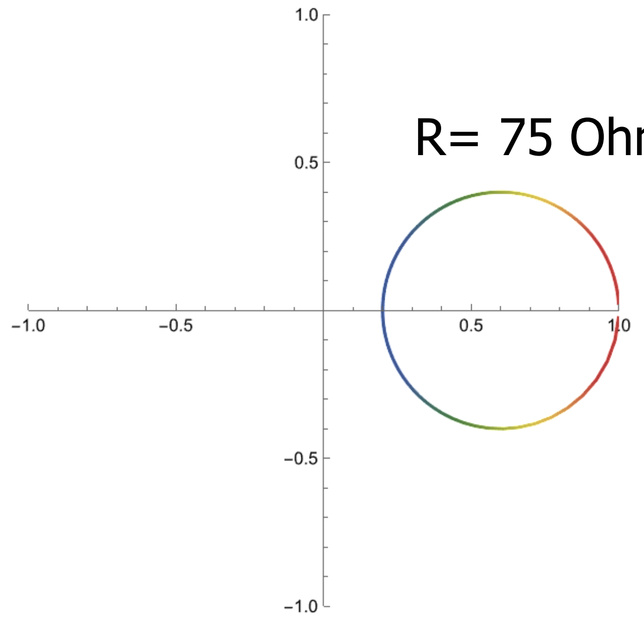


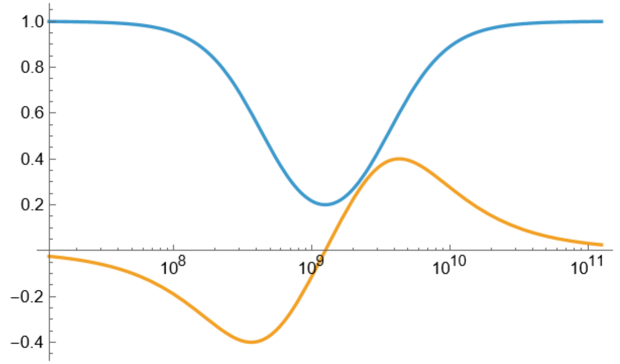
R= 50 Ohms



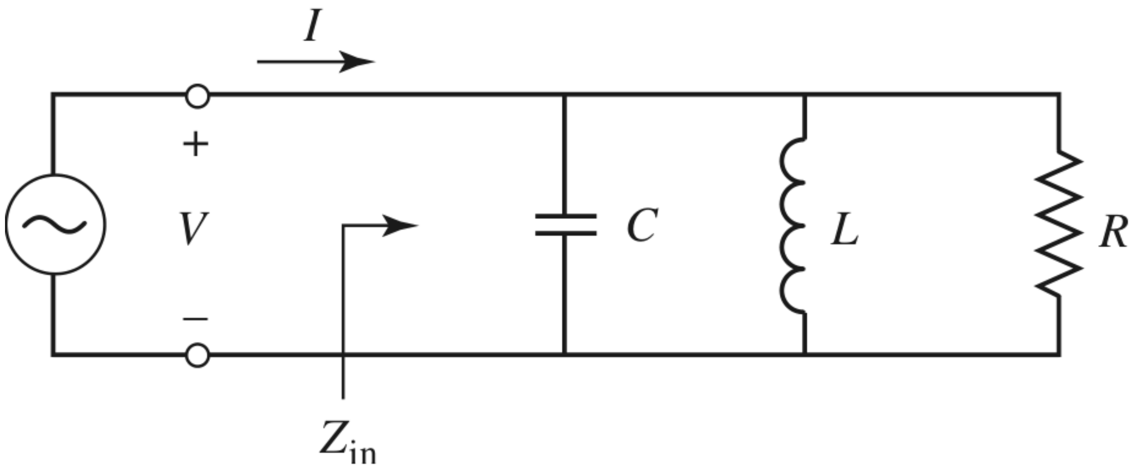


R= 75 Ohms

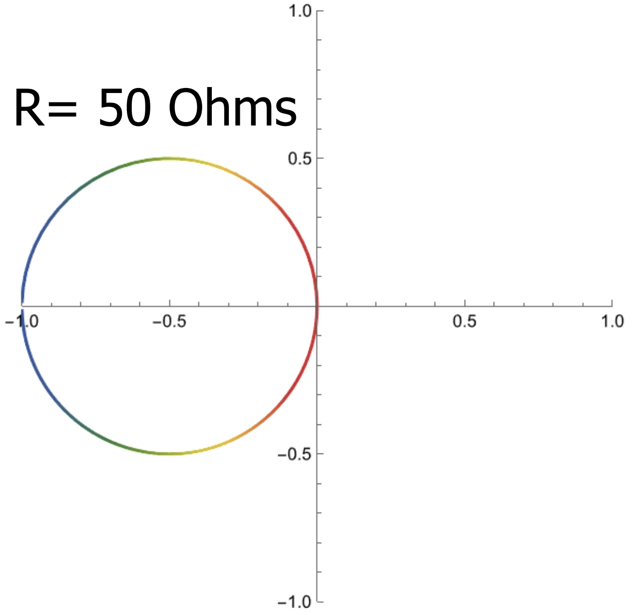


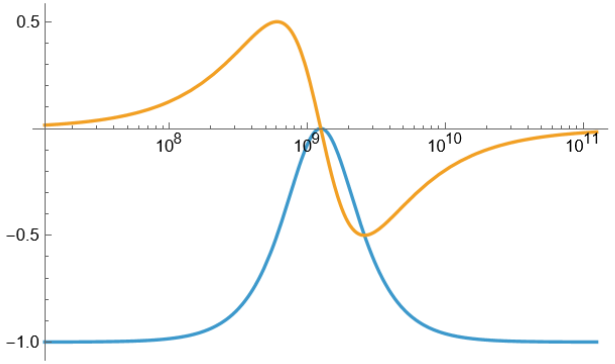


Parallel Resonant Circuit on the Smith chart

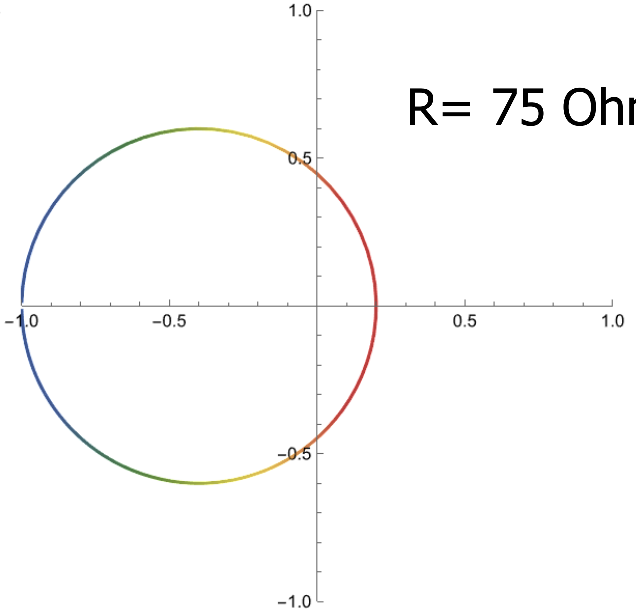


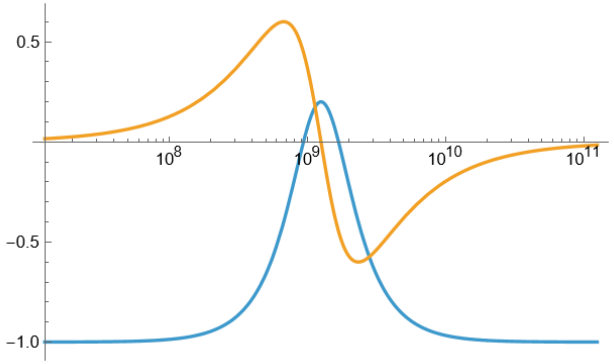
R= 50 Ohms





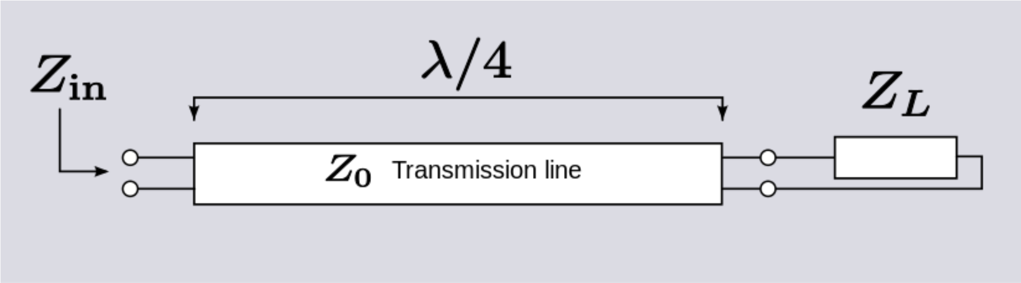
R= 75 Ohms



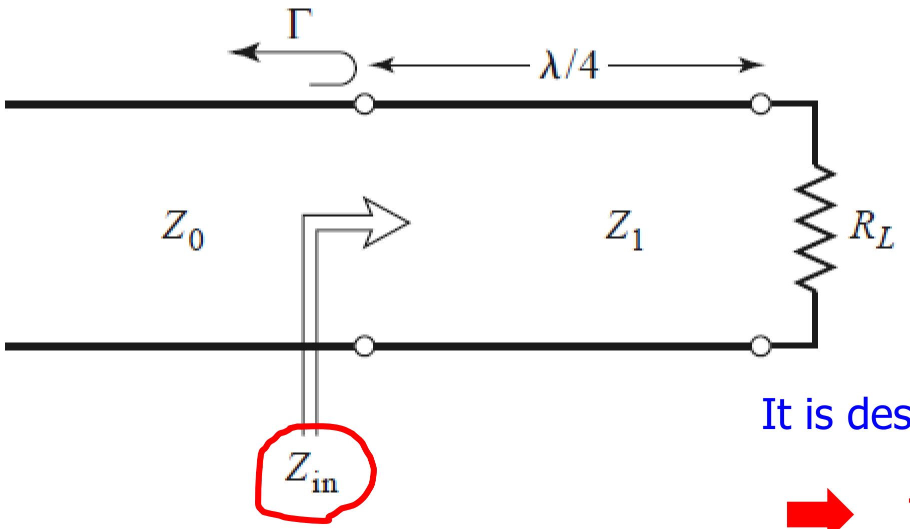


The Quarter-wave transformer

- Quarter-wave transformer (QWR) is useful for impedance matching.
- QWR has a pre-determined length of $\lambda/4$, and the termination is designed to produce the required impedance.



Consider



R_L: load line impedance (real, known)
Z_0: feedline characteristic impedance (real, known)

QWR: lossless piece of TL of unknown characteristic impedance Z_1 and length $\lambda/4$.

It is desired to match the load to the Z_0 line by using $\lambda/4$ section of the line

➡ To make $\Gamma=0$ when looking into $\lambda/4$ matching section!

The Quarter-wave transformer

- The input impedance (earlier) $Z_{\text{in}} = Z_1 \dfrac{R_L + jZ_1 \tan\beta\ell}{Z_1 + jR_L \tan\beta\ell}$ for $\beta\ell = (2\pi/\lambda)(\lambda/4) = \pi/2$

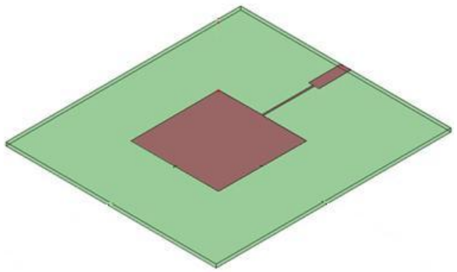
$$\boxed{Z_{\text{in}} = \frac{Z_1^2}{R_L}}$$

To have $\Gamma = 0$ ⟹ $Z_{\text{in}} = Z_0$

$\therefore$ $\boxed{Z_1 = \sqrt{Z_0 R_L}}$: geometric mean of the load and source impedance

→ Perfect match may be achieved at one frequency. But mismatch will occur at other frequencies

→ SWR=1 (meaning that no standing wave on the feedline. But will be standing waves on the λ/4 matching section.)



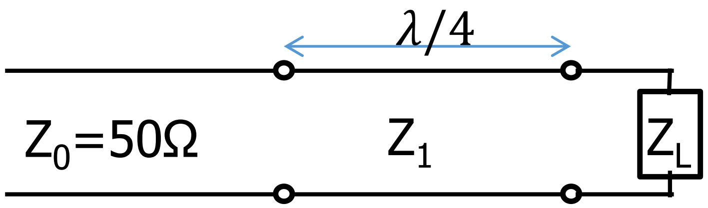
← Microstrip patch antenna

The Quarter-wave transformer

Example: Frequency response of a quarter-wave transformer

Consider a load resistance $R_L = 100\ \Omega$ to be matched to a 50 Ω line with a quarter-wave transformer. Find the characteristic impedance of the matching section and plot the magnitude of the reflection coefficient versus normalized frequency, f/f_o, where f_o is the frequency at which the line is $\lambda/4$ long.

Solution:



$$Z_1 = \sqrt{Z_0 R_L} = \sqrt{(50)(100)} = 70.71\ \Omega$$

$$|\Gamma| = \left|\frac{Z_{\rm in} - Z_0}{Z_{\rm in} + Z_0}\right| \quad \text{and} \quad Z_{\rm in} = Z_1 \frac{R_L + jZ_1 \tan\beta\ell}{Z_1 + jR_L \tan\beta\ell}$$

$$\beta\ell = \left(\frac{2\pi}{\lambda}\right)\left(\frac{\lambda_0}{4}\right) = \left(\frac{2\pi f}{v_p}\right)\left(\frac{v_p}{4f_o}\right) = \frac{\pi f}{2f_o}$$

For f=f_0, $\beta\ell = \pi/2$