Previous studies and VSR

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Conventional magnet and Schwartz transformation

From Maxwell's equations in free space

$$\nabla \times \mathbf{B} = \nabla \times (\nabla \times \mathbf{A}) = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2\mathbf{A} = 0 \quad \xrightarrow{\text{Lorentz gauge } \nabla\cdot\mathbf{A}=0} \quad \nabla^2\mathbf{A} = 0\,,$$

$$\nabla \cdot \mathbf{B} = \nabla \cdot (-\nabla\Phi) = -\nabla^2\Phi = 0 \quad \rightarrow \quad \nabla^2\Phi = 0\,.$$

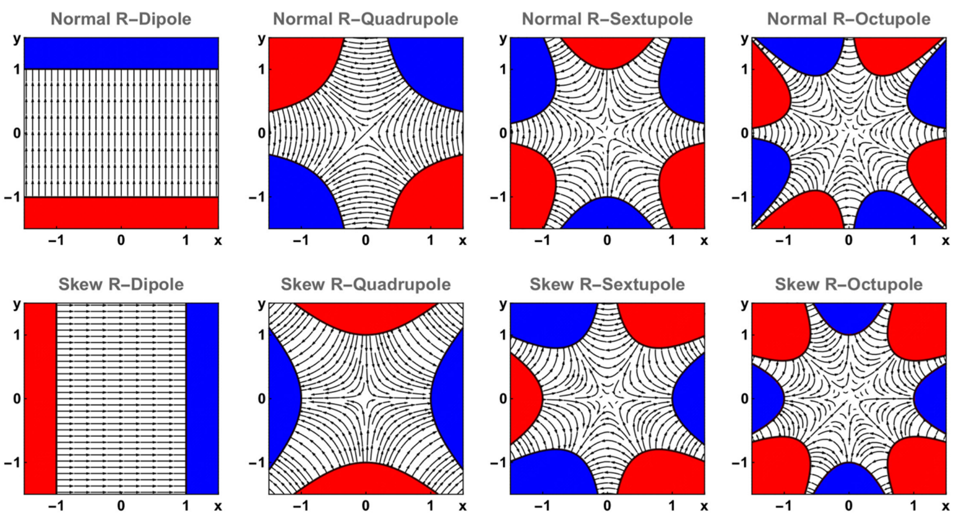
and assuming the transverse magnetic field $\mathbf{B} = \left(B_x, B_y, 0\right)$ and $\mathbf{A} = (0, 0, A_z)$ s.t. $B_x = \frac{\partial A_z}{\partial y}$ and $B_y = -\frac{\partial A_z}{\partial x}$, we can solve the Laplace's equation for scalar potential with proper boundary condition in polar coordinates as

$$\Phi(r,\theta) = \sum_{n=1}^{\infty} r^n(a_n \cos n\theta + b_n \sin n\theta) = \sum_{n=1}^{\infty} \Phi_n \quad \cdots (1)\,.$$

For each eigenmodes, we call *normal* $2n$ −pole magnet when $a_n = 0$ and *skew* $2n$ − pole magnet when $b_n = 0$. The only difference between normal magnet and skew magnet is rotation of $\frac{\pi}{2n}$ rad, since

$$a_n r^n \cos n\theta = a_n r^n \sin\left(n\theta + \frac{\pi}{2}\right) = a_n r^n \sin n\left(\theta + \frac{\pi}{2n}\right) \quad \cdots (2)\,.$$

Conventional magnet and Schwartz transformation



$$\Phi_{n,\text{normal}} = b_n r^n \sin n\theta = a_n r^n \sin n\theta\,,$$

$$\Phi_{n,\text{skew}} = a_n r^n \cos n\theta$$

$$\mathbf{B}_n = -n a_n r^{n-1}\left(\sin n\theta\,\hat{\mathbf{r}} \pm \cos n\theta\,\hat{\boldsymbol{\theta}}\right)$$

$$\equiv B_n\left(\sin n\theta\,\hat{\mathbf{r}} \pm \cos n\theta\,\hat{\boldsymbol{\theta}}\right),$$

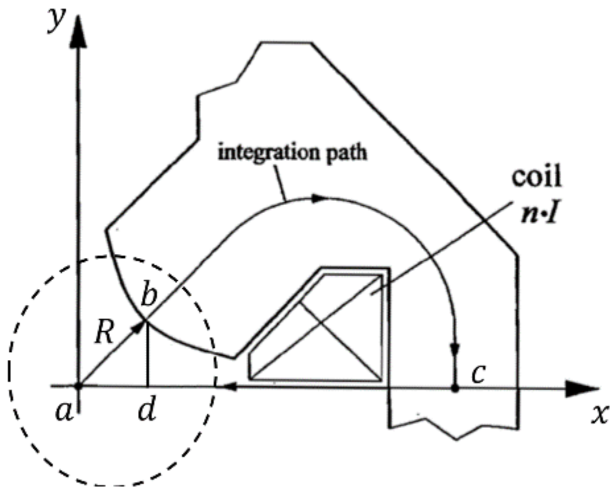
Take derivatives of B_ns until constant : $\quad g^{(n-1)} \equiv \dfrac{\partial^{n-1} B_n}{\partial r^{n-1}} = -n!\,a_n$

e.g. Taylor expansion of B_y on the $x-$axis :

$$B_y = B_y\Big|_{y=0} + \frac{\partial B_y}{\partial x}\Big|_{y=0} x + \frac{1}{2!}\frac{\partial^2 B_y}{\partial x^2}\Big|_{y=0} x^2 + \frac{1}{3!}\frac{\partial^3 B_y}{\partial x^3}\Big|_{y=0} x^3 + O(x^4)$$

$$= B_0 + gx + \frac{1}{2}g' x^2 + \frac{1}{6}g'' x^3 + O(x^4)\,.$$

Conventional magnet and Schwartz transformation



(Appendix) Derivation of g $(n \geq 2)$

Ampere's law for $a \rightarrow b \rightarrow c \rightarrow a$:

$$\oint \mathbf{H} \cdot d\mathbf{l} = \int_a^b \mathbf{H}_0 \cdot d\mathbf{l} + \int_b^c \mathbf{H}_{\text{Fe}} \cdot d\mathbf{l} + \int_c^a \mathbf{H} \cdot d\mathbf{l} = \frac{1}{\mu_0} \int_a^b \mathbf{B} \cdot d\mathbf{l} = NI \, ,$$

Ampere's law for $a \rightarrow b \rightarrow d \rightarrow a$:

$$\oint \mathbf{B} \cdot d\mathbf{l} = \int_a^b \mathbf{B} \cdot d\mathbf{l} + \int_b^d B_y dy + \int_d^a B_x dx = 0$$

$$\rightarrow \int_a^b \mathbf{B} \cdot d\mathbf{l} = -\int_d^a B_x dx - \int_b^d B_y dy = \int_a^d B_x dx + \int_d^b B_y dy = \int_0^{R\cos\frac{\pi}{2n}} B_x\Big|_{y=0} dx + \int_0^{R\sin\frac{\pi}{2n}} B_y\Big|_{x=d} dy$$

$$\therefore \int_0^{R\cos\frac{\pi}{2n}} B_x\Big|_{y=0} dx + \int_0^{R\sin\frac{\pi}{2n}} B_y\Big|_{x=R\cos\frac{\pi}{2n}} dy = \mu_0 NI$$

e.g. Quadrupole magnet $(n = 2)$

$$\mu_0 NI = \int_0^{\frac{1}{\sqrt{2}}R} gy\Big|_{y=0} dx + \int_0^{\frac{1}{\sqrt{2}}R} gx\Big|_{x=\frac{1}{\sqrt{2}}R} dy = \frac{1}{2} g R^2 \qquad \therefore \; g = \frac{2\mu_0 nI}{R^2}$$

Conventional magnet and Schwartz transformation

We can show that infinitesimal complex variables $dz = dx + idy$ and $dz^* = dx - idy$ are linearly independent, thereby the derivative of an analytic function $f(z) = f(x + iy)$ can be defined as

$$df = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy = \frac{1}{2}\left(\frac{\partial f}{\partial x} - i\frac{\partial f}{\partial y}\right)dz + \frac{1}{2}\left(\frac{\partial f}{\partial x} + i\frac{\partial f}{\partial y}\right)dz^* \equiv \frac{\partial f}{\partial z}dz + \frac{\partial f}{\partial z^*}dz^*,$$

which means that $\frac{\partial}{\partial z} = \frac{1}{2}\frac{\partial}{\partial x} + \frac{1}{2i}\frac{\partial}{\partial y}$ and $\frac{\partial}{\partial z^*} = \frac{1}{2}\frac{\partial}{\partial x} - \frac{1}{2i}\frac{\partial}{\partial y}$.

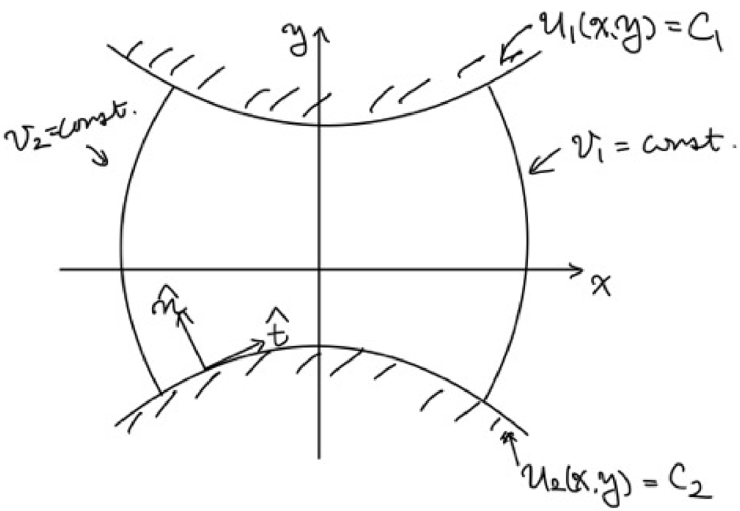
The condition for being an analytic function is $\frac{\partial f}{\partial z^*} = 0$, which is identical with

$Cauchy - Riemann$ equations $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$ and $\frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y}$ for $f(z) = u(x, y) + iv(x, y)$.

Conventional magnet and Schwartz transformation

Take $f \longleftarrow \frac{\partial f}{\partial z}$ to get $\frac{\partial}{\partial z^*}\frac{\partial f}{\partial z} = \frac{1}{4}\nabla^2 f = 0$ and from $\nabla^2 f = \nabla^2 u + i\nabla^2 v$, we can say that any analytic function satisfies the Laplace′s equation, with $\nabla^2 u = \nabla^2 v = 0$.

Also, we can show that ∇u ∇v are orthogonal by using Cauchy − Riemann equations, therefore *any analytic function corresponds to a solution of two − dimensional boundary problem of Dirichlet type*.



Assume that $v(x,y)$ is the magnetic scalar potential,

$$\frac{df}{dz} = \left(\frac{1}{2}\frac{\partial}{\partial x} + \frac{1}{2i}\frac{\partial}{\partial y}\right)(u + iv) = \frac{\partial v}{\partial y} + i\frac{\partial v}{\partial x} = -B_y - iB_x$$

Therefore,

$$B_x + iB_y = -i\left(\frac{df}{dz}\right)^* \quad \cdots (3)$$

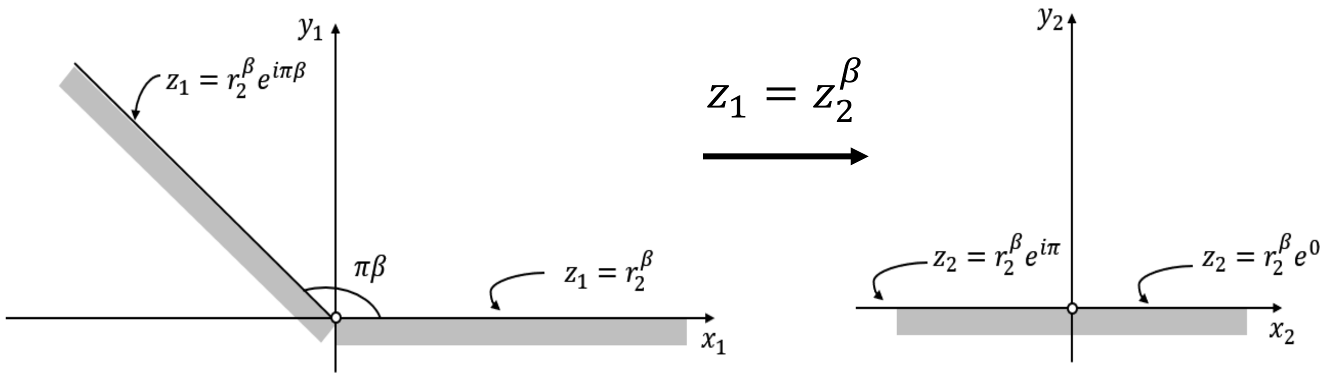
Conventional magnet and Schwartz transformation

We can find that the physical meaning of $u(x, y)$ is the vector potential A_z, from

$$B_x = -\frac{\partial v}{\partial x} = \frac{\partial u}{\partial y} = \frac{\partial A_z}{\partial y} \text{ and } B_x = -\frac{\partial v}{\partial x} = \frac{\partial u}{\partial y} = \frac{\partial A_z}{\partial y} .$$

Therefore we can define the *complex potential* f as $f = A_z + i\Phi_m \quad \cdots (4)$.

From the fact that any analytic complex transformation including the finite number of singularity preserves the angle (*conformal*), we can map the solution space of Laplace's equation $\nabla^2 f = 0$ into a upper half of new complex plane.

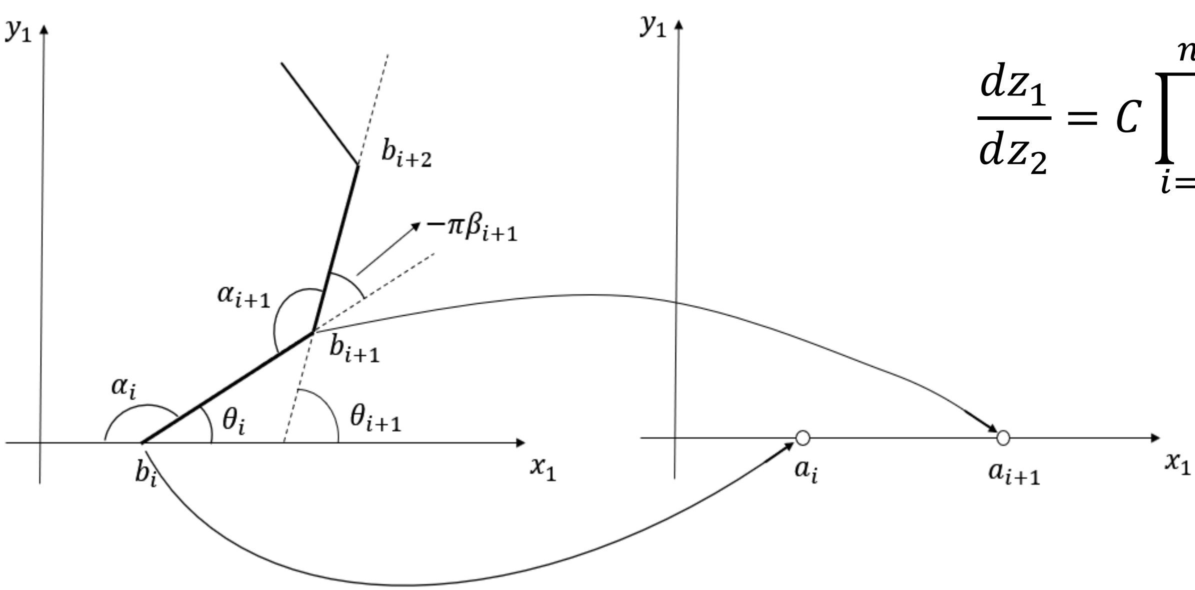


Conventional magnet and Schwartz transformation

We know that the magnetic field is constant in y −direction, so it is easy to find potentials in z_2 plane: $\Phi_m = -By$ and $A_z = -Bx$ (from Cauchy-Riemann equations).

Therefore the complex potential is $f = -B(x + iy) = -Bz$.

The generalization of previous transformation from z_1 plane to z_2 plane is known as $Schwartz\ transformation$.



$$\frac{dz_1}{dz_2} = C\prod_{i=1}^{n}(z_2 - a_i)^{\frac{\alpha_i}{\pi}-1}$$

Conventional magnet and Schwartz transformation

For one 'cell' of conventional $2n-$pole magnet, the Schwartz transformation becomes

$$\frac{dz_1}{dz_2} = Cz_2^{\frac{\alpha}{\pi}-1} = Cz_2^{\frac{1}{n}-1} \qquad \therefore z_2 = \left(\frac{1}{nC}\right)^n z_1^n .$$

Therefore, the complex potential f which provides a homogeneous field in z_2 plane

can be described in z_1 plane as $f = -Bz_2 = -\frac{B}{(nC)^n} z_1^n \equiv C_n z_1^n \quad \cdots (5).$

From Eq. (2), we can find the rotation transformation $z \to z_1 = ze^{i\frac{\pi}{2n}}$ for the skew

multipole magnet. Hence $f_{\text{skew}} = C_n z_1^n = iC_n z^n \quad \cdots (6)$

We can get the magnetic field, scalar and vector potential by Eqs. (3) and (4).

Conventional magnet and Schwartz transformation

e. g. Quadrupole magnets ($n = 2$)

(a) Normal magnet

$$f = -Bz_2 = C_2 z_1^2 = C_2(x_1 + iy_1)^2 = C_2(x_1^2 - y_1^2 + 2ix_1y_1)$$

$$\Phi_m = \mathrm{Im}\, f = 2C_2x_1y_1, \qquad A_z = \mathrm{Re}\, f = C_2(x_1^2 - y_1^2)$$

$$B_x + iB_y = -i\left(\frac{df}{dz_1}\right)^* = -2iC_2z_1^* = -2iC_2(x_1 - iy_1) = -2C_2(y_1 + ix_1), \qquad g = \frac{\partial B_y}{\partial x} = -2C_2$$

(b) Skew magnet

$$f_{\text{skew}} = iC_2z^2 = iC_2(x^2 - y^2 + 2ixy) = gxy - i\frac{1}{2}g(x^2 - y^2)$$

$$\Phi_m = \mathrm{Im}\, f_{\text{skew}} = -\frac{1}{2}g(x^2 - y^2), \qquad A_z = \mathrm{Re}\, f_{\text{skew}} = gxy$$

$$B_x + iB_y = -i\left(\frac{df_{\text{skew}}}{dz}\right)^* = -i(2iC_2z)^* = gz^* = gx - igy$$

Conventional magnet and Schwartz transformation

The scalar potential in polar coordinate Eq. (1) can be expressed by writing $z = re^{i\theta}$ and $C_n = b_n + ia_n$ so that $f = (b_n + ia_n)r^n e^{in\theta}$ and $\Phi_m = \mathrm{Im}\, f = r^n(a_n \cos n\theta + b_n \sin n\theta)$.

We can also find the vector potential *without any calculation*,

$A_z = \mathrm{Re}\, f = r^n(b_n \cos n\theta - a_n \sin n\theta)$.

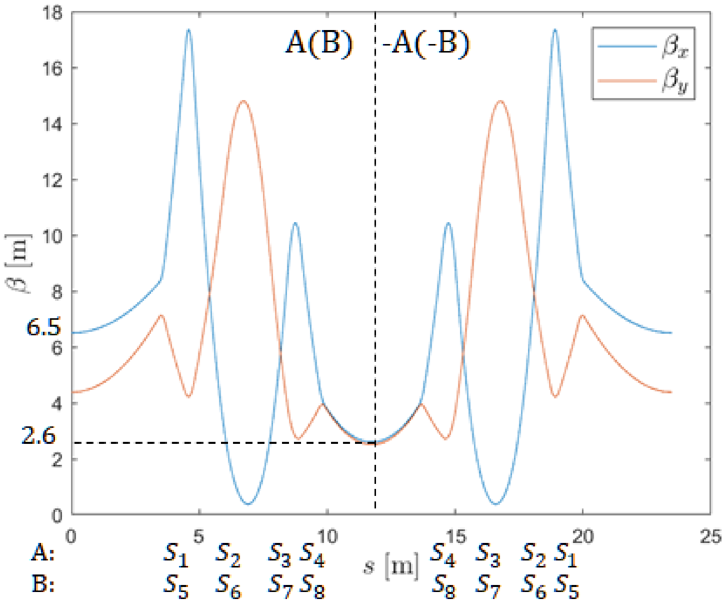
This is because the vector potential = stream function in two dimenstion space.

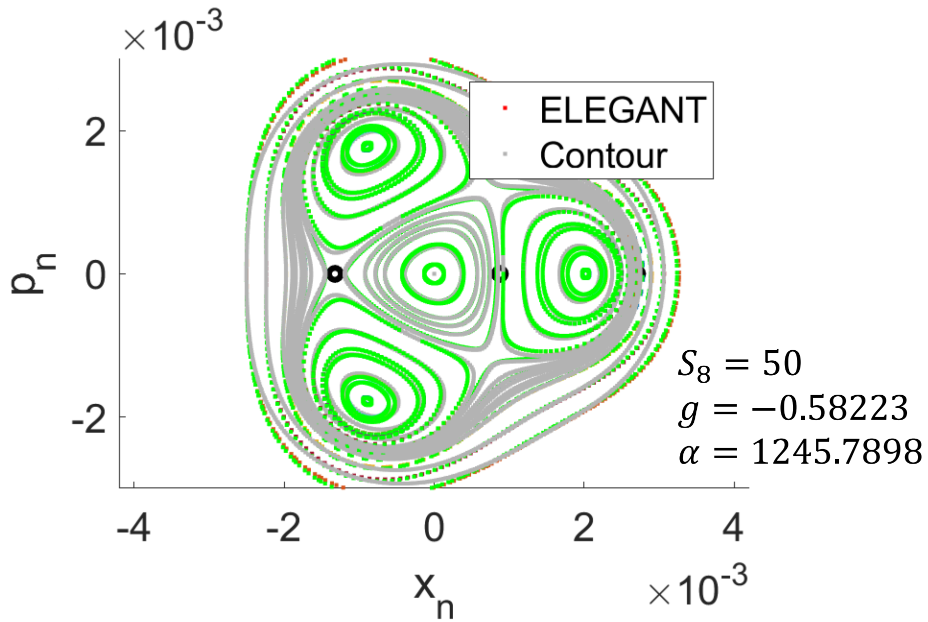
- New type of magnet – multiple angle magnets?

Symmetry Breaking in Transverse Beam Dynamics

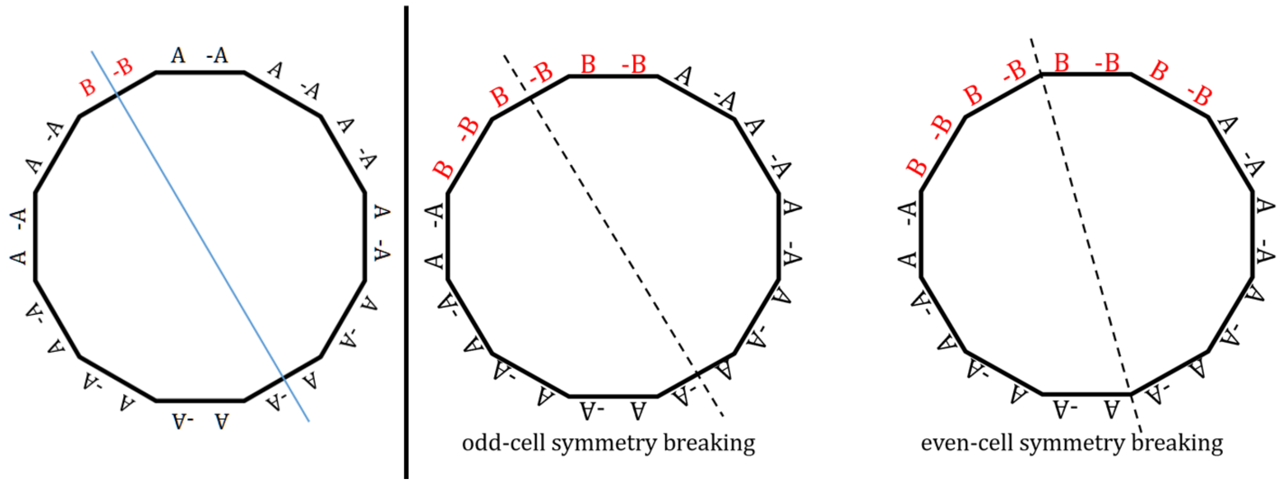
- Included in Dr. Ki Moon Nam's study shortly.

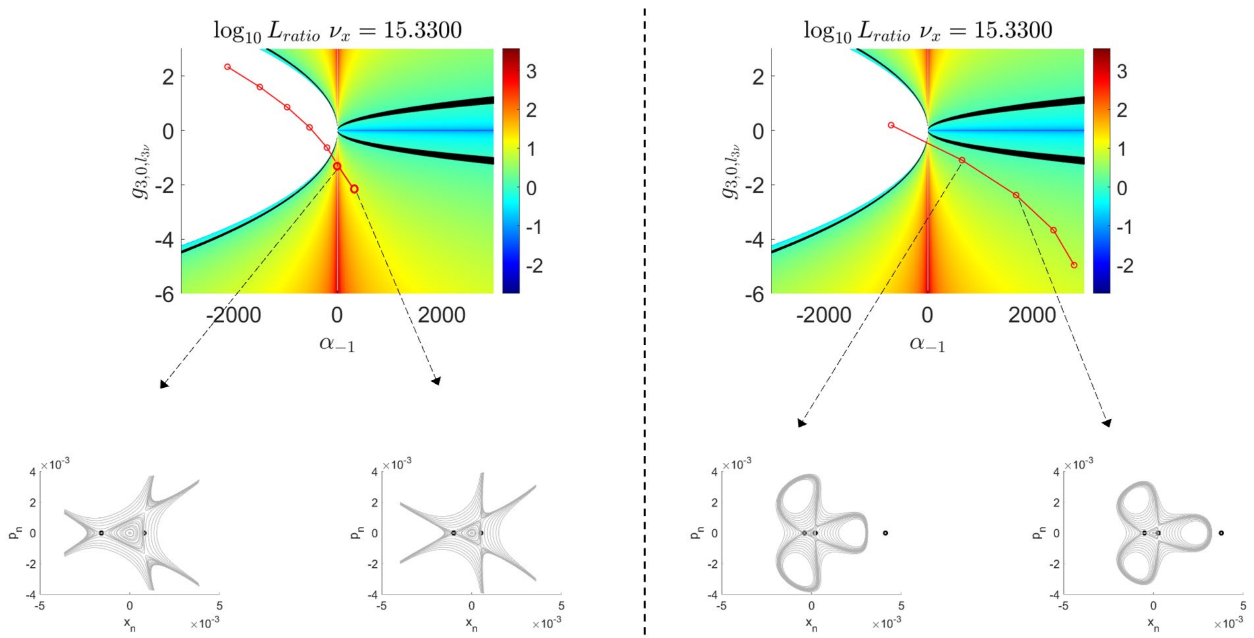
- Focused on the symmetry breaking via sextupole magnets and multiple-cell symmetry breaking, for possible set of β_{x0} found by Mr. Youngmin Park.





Symmetry Breaking in Transverse Beam Dynamics





- Single-cell S.B.
- Double-cell S.B.

RF phase and voltage modulation

1. RF phase modulation

The longitudinal Hamiltonian in normalied phase space $\left(\phi, \mathcal{P} = -\frac{h|\eta|}{\nu_s}\delta\right)$ with stationary synchrotron motion, i. e. $\phi_s = 0$ is

$$\mathcal{H}_0 = \frac{1}{2}\omega_0\nu_s\mathcal{P}^2 + \omega_0\nu_s(1 - \cos\phi) \equiv \omega_0\widehat{\mathcal{H}}_0\,.$$

Consider the RF phase modulation as perturbation, i. e. $\widehat{\mathcal{H}} = \widehat{\mathcal{H}}_0 + \widehat{\mathcal{H}}_1$,

$$\begin{aligned}
\widehat{\mathcal{H}}_1 &= \nu_m a\mathcal{P}\cos(\nu_m\theta + \chi_0) \\
&= \nu_m a\left[(2J)^{\frac{1}{2}}\cos\psi + \frac{(2J)^{\frac{3}{2}}}{64}\cos 3\psi + \cdots\right]\cos(\nu_m\theta + \chi_0) \\
&= \nu_m a\sqrt{\frac{J}{2}}\left[\cos(\psi + \nu_m\theta + \chi_0) + \cos(\psi - \nu_m\theta - \chi_0)\right] \\
&\quad + \nu_m a\frac{(2J)^{\frac{3}{2}}}{128}\left[\cos(3\psi + \nu_m\theta + \chi_0) + \cos(3\psi - \nu_m\theta - \chi_0)\right] + \cdots
\end{aligned}$$

Therefore the RF phase error only generates the odd order of parametric resonance.

RF phase and voltage modulation

$$\therefore\ \hat{\mathcal{H}} \approx \nu_s J - \frac{1}{16}\nu_s J^2 + \sum_{k=0}^{\infty} \nu_m f_{2k+1} J^{k+\frac{1}{2}} \{\cos[(2k+1)\psi + \nu_m\theta + \chi_0] + \cos[(2k+1)\psi + \nu_m\theta + \chi_0]\},$$

where $f_1 = \dfrac{a}{\sqrt{2}}, f_2 = \dfrac{a}{32\sqrt{2}}, \cdots.$

If phase modulation amplitude a is small enough, the dominant resonance is *Dipole mode*, which corresponds to the $k = 0$ mode and $\nu_m \approx \nu_s$.

Transform the Hamiltonian of Dipole mode $\hat{\mathcal{H}} \approx \nu_s J - \frac{1}{16}\nu_s J^2 + \frac{\nu_s a}{\sqrt{2}} J^{\frac{1}{2}} \cos(\psi - \nu_m - \chi_0)$ into the resonance rotating frame (χ, I) by the generating function $F_2(\psi, I) = (\psi - \nu_m\theta - \chi_0 - \pi)I$

$$J = \frac{\partial F_2}{\partial \psi} = I, \qquad \chi = \frac{\partial F_2}{\partial I} = \psi - \nu_m\theta - \chi_0 - \pi,$$

$$\hat{K} = \hat{\mathcal{H}} + \frac{\partial F_2}{\partial \theta} = (\nu_s - \nu_m)I - \frac{1}{16}\nu_s I^2 + \nu_s \frac{a}{\sqrt{2}} I^{\frac{1}{2}} \cos\chi$$

RF phase and voltage modulation

Fixed points can be obtained by solving $g^3 - 16\left(1 - \frac{\nu_\mathrm{m}}{\nu_\mathrm{s}}\right) g + 8a = 0$, where $\chi = 0, \pi$ and $g = \sqrt{2I}\cos\chi$.

When $\nu_\mathrm{m} \le \nu_\mathrm{bif} = \nu_\mathrm{s}\left[1 - \frac{3}{16}(4a)^{\frac{2}{3}}\right]$: *bifurcation tune* ,

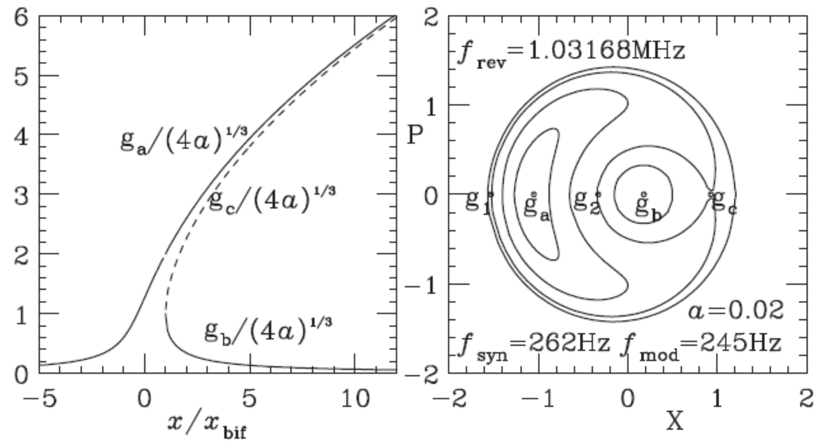
$$g_a(x) = -\frac{8}{\sqrt{3}} x^{\frac{1}{2}} \cos\frac{\xi}{3}, \qquad g_b(x) = \frac{8}{\sqrt{3}} x^{\frac{1}{2}} \sin\left(\frac{\pi}{6} - \frac{\xi}{3}\right) \quad \text{and} \quad g_c(x) = \frac{8}{\sqrt{3}} x^{\frac{1}{2}} \sin\left(\frac{\pi}{6} + \frac{\xi}{3}\right)$$

where $x = 1 - \frac{\nu_\mathrm{m}}{\nu_\mathrm{s}}$, $x_\mathrm{bif} = 1 - 1 - \frac{\nu_\mathrm{bif}}{\nu_\mathrm{s}} = \frac{3}{16}(4a)^{\frac{2}{3}}$, and $\xi = \tan^{-1}\left(\sqrt{\left(\frac{x}{x_\mathrm{bif}}\right)^3 - 1}\right)$.

Otherwise when $\nu_\mathrm{m} > \nu_\mathrm{bif}$,

$$g_a(x) = -(4a)^{\frac{1}{3}}\left[\left(\sqrt{1 - \left(\frac{x}{x_\mathrm{bif}}\right)^3} + 1\right)^{\frac{1}{3}} - \left(\sqrt{1 - \left(\frac{x}{x_\mathrm{bif}}\right)^3} - 1\right)^{\frac{1}{3}}\right]$$

RF phase and voltage modulation



The island tune

$$\nu_{\text{island}} = \left|\frac{\nu_s a}{2g}\right|\left(1-\frac{g^3}{4a}\right)^{\frac{1}{2}} = \left|\nu_s\left(1-\frac{g^2}{16}\right)-\nu_m\right|\left(1-\frac{g^3}{4a}\right)^{\frac{1}{2}}$$

RF phase and voltage modulation

2. RF voltage modulation

When $V \to V + \Delta V, \nu_s \to \nu_s \left(1 + \frac{\Delta V}{V}\right)^{\frac{1}{2}} \approx \nu_s \left(1 + \frac{\Delta V}{2V}\right) \equiv \nu_s [1 + b \sin(\nu_m \theta + \chi)]$.

The perturbation Hamiltonian $\widehat{\mathcal{H}}_1$ is then $\widehat{\mathcal{H}}_1 = \nu_s b \sin(\nu_m \theta + \chi)(1 - \cos\phi)$.

Let $\chi = 0$ for convinience, then

$$\widehat{\mathcal{H}}_1 = \nu_s b \sin(\nu_m \theta) \sum_{n=-\infty}^{\infty} G_n(J) e^{in\psi} = \nu_s b \sin(\nu_m \theta) \sum_{n=-\infty}^{\infty} |G_n(J)| e^{i(n\psi + \gamma_n)}$$

$$= \nu_s b \left\{ |G_0(J)| \sin(\nu_m \theta) + \sum_{n=1}^{\infty} [|G_n(J)| \sin(\nu_m \theta + n\psi + \gamma_n) + |G_n(J)| \sin(\nu_m \theta - n\psi - \gamma_n)] \right\}$$

$$= \nu_s b \sum_{n=-\infty}^{\infty} |G_n(J)| \sin(\nu_m \theta - n\psi - \gamma_n) \ .$$

$$\therefore \widehat{\mathcal{H}} = \nu_s J - \frac{1}{16} \nu_s J^2 + \nu_s b \sum_{n=-\infty}^{\infty} |G_n(J)| \sin(\nu_m \theta - n\psi - \gamma_n)$$

RF phase and voltage modulation

Transform into the resonance rotating frame $(\tilde{\psi},\tilde{J})$ with $F_2(\psi,\tilde{J}) = \left(\psi - \frac{\nu_\mathrm{m}}{k}\theta + \frac{\gamma_k}{k} + \frac{\pi}{2k}\right)\tilde{J}$:

$$\tilde{J} = \frac{\partial F_2}{\partial \psi} = J, \qquad \tilde{\psi} = \frac{\partial F_2}{\partial \tilde{J}} = \psi - \frac{\nu_\mathrm{m}}{k}\theta + \frac{\gamma_k}{k} + \frac{\pi}{2k},$$

$$\hat{K} = \hat{\mathcal{H}} + \frac{\partial F_2}{\partial \theta} = \nu_\mathrm{s}\tilde{J} - \frac{1}{16}\nu_\mathrm{s}\tilde{J}^2 + \nu_\mathrm{s} b\left|G_k(\tilde{J})\right| \sin\left(k\tilde{\psi} - \frac{\pi}{2}\right) - \frac{\nu_\mathrm{m}}{k}\tilde{J} + \Delta\hat{\mathcal{K}}\left(\tilde{J},\tilde{\psi},\theta\right)$$

$$= \left(\nu_\mathrm{s} - \frac{\nu_\mathrm{m}}{k}\right)\tilde{J} - \frac{1}{16}\nu_\mathrm{s}\tilde{J}^2 + \nu_\mathrm{s} b\left|G_k(\tilde{J})\right| \cos k\tilde{\psi} + \Delta\hat{\mathcal{K}}\left(\tilde{J},\tilde{\psi},\theta\right)$$

Take average with respect to θ gives $\langle\hat{\mathcal{K}}\rangle = \left(\nu_\mathrm{s} - \frac{\nu_\mathrm{m}}{k}\right)\tilde{J} - \frac{1}{16}\nu_\mathrm{s}\tilde{J}^2 + \nu_\mathrm{s} b\left|G_k(\tilde{J})\right| \cos k\tilde{\psi}$.

The resonance strength is the strongest in the lowest harmonic for a small action particle, since $|G_{n+2}/G_n| \sim J$. Thereby the $k = 2$ mode is dominant, which is *Quadrupole mode*.

RF phase and voltage modulation

Transform into the resonance rotating frame $(\tilde{\psi}, \tilde{J})$ with $F_2(\psi, \tilde{J}) = \left(\psi - \frac{\nu_\mathrm{m}}{k}\theta + \frac{\gamma_k}{k} + \frac{\pi}{2k}\right)\tilde{J}$:

$$\tilde{J} = \frac{\partial F_2}{\partial \psi} = J, \qquad \tilde{\psi} = \frac{\partial F_2}{\partial \tilde{J}} = \psi - \frac{\nu_\mathrm{m}}{k}\theta + \frac{\gamma_k}{k} + \frac{\pi}{2k},$$

$$\hat{K} = \hat{\mathcal{H}} + \frac{\partial F_2}{\partial \theta} = \nu_\mathrm{s}\tilde{J} - \frac{1}{16}\nu_\mathrm{s}\tilde{J}^2 + \nu_\mathrm{s} b\left|G_k(\tilde{J})\right| \sin\left(k\tilde{\psi} - \frac{\pi}{2}\right) - \frac{\nu_\mathrm{m}}{k}\tilde{J} + \Delta\hat{\mathcal{K}}\left(\tilde{J}, \tilde{\psi}, \theta\right)$$

$$= \left(\nu_\mathrm{s} - \frac{\nu_\mathrm{m}}{k}\right)\tilde{J} - \frac{1}{16}\nu_\mathrm{s}\tilde{J}^2 + \nu_\mathrm{s} b\left|G_k(\tilde{J})\right| \cos k\tilde{\psi} + \Delta\hat{\mathcal{K}}\left(\tilde{J}, \tilde{\psi}, \theta\right)$$

Take average with respect to θ gives $\langle\hat{\mathcal{K}}\rangle = \left(\nu_\mathrm{s} - \frac{\nu_\mathrm{m}}{k}\right)\tilde{J} - \frac{1}{16}\nu_\mathrm{s}\tilde{J}^2 + \nu_\mathrm{s} b\left|G_k(\tilde{J})\right| \cos k\tilde{\psi}$.

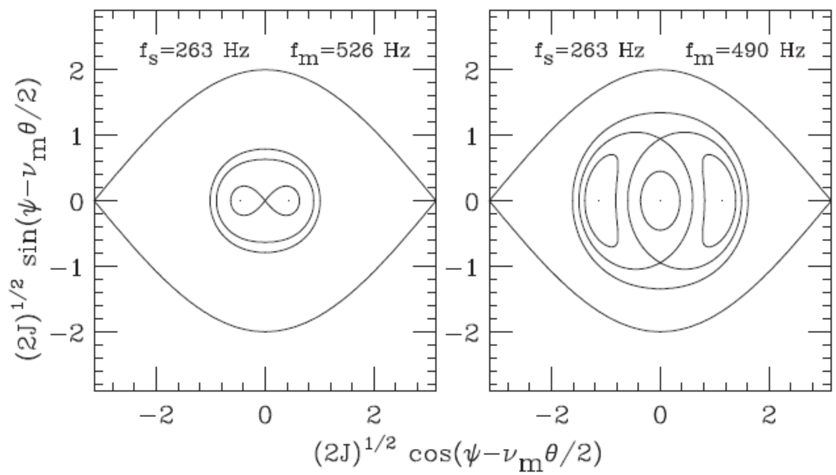
The resonance strength is the strongest in the lowest harmonic for a small action particle, since $|G_{n+2}/G_n| \sim J$. Thereby the $k = 2$ mode is dominant, which is $Quadrupole\ mode$.

RF phase and voltage modulation

Fixed points for Quadrupole mode

$$\cos 2\psi_0 = 1 \;:\; J_{\mathrm{SFP}} = \begin{cases} 8\left(1-\dfrac{\nu_m}{2\nu_s}\right)+2b & \text{if } \nu_m < 2\nu_s + \dfrac{b}{2}\nu_s \\ 0 & \text{if } \nu_m \geq 2\nu_s + \dfrac{b}{2}\nu_s \end{cases}$$

$$\cos 2\psi_0 = -1 \;:\; J_{\mathrm{UFP}} = \begin{cases} 8\left(1-\dfrac{\nu_m}{2\nu_s}\right)-2b & \text{if } \nu_m < 2\nu_s - \dfrac{b}{2}\nu_s \\ 0 & \text{if } 2\nu_s - \dfrac{b}{2}\nu_s \leq \nu_m \leq 2\nu_s + \dfrac{b}{2}\nu_s \end{cases}$$



RF phase and voltage modulation

Island tune

$$\nu_{\text{island}} = \frac{\pi \nu_s \sqrt{2bJ_{\text{SFP}}}}{K(k)} x^{\frac{1}{4}}, \qquad k = \frac{1}{\sqrt{2}} \sqrt{1 - \frac{xJ_{\text{SFP}} - J_{\text{UFP}}}{\sqrt{x}(J_{\text{SFP}} - J_{\text{UFP}})}}$$

3. Phase modulation can increase the beam lifetime

PHYSICAL REVIEW SPECIAL TOPICS - ACCELERATORS AND BEAMS, VOLUME **3**, 050701 (2000)

Improvement in the beam lifetime by means of an rf phase modulation at the KEK Photon Factory storage ring

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In the 2.5-GeV Photon Factory storage ring at KEK, we have found that the beam lifetime can be improved by modulating the phase of an rf accelerating voltage at a frequency of 2 times the synchrotron oscillation frequency. By applying this phase modulation with a peak-to-peak amplitude of 3.2°, the beam lifetime could be improved, typically, from 22 to 36 h under a beam current of about 360 mA. At the same time, the longitudinal coupled-bunch instability could be considerably suppressed. The improvement in the beam lifetime can be explained as an improved Touschek lifetime, which was caused by a quadrupole-mode longitudinal oscillation of the stored bunches.

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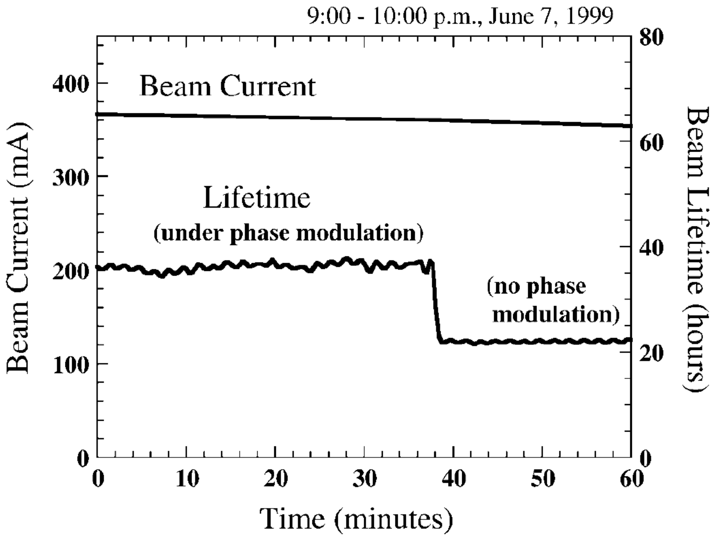
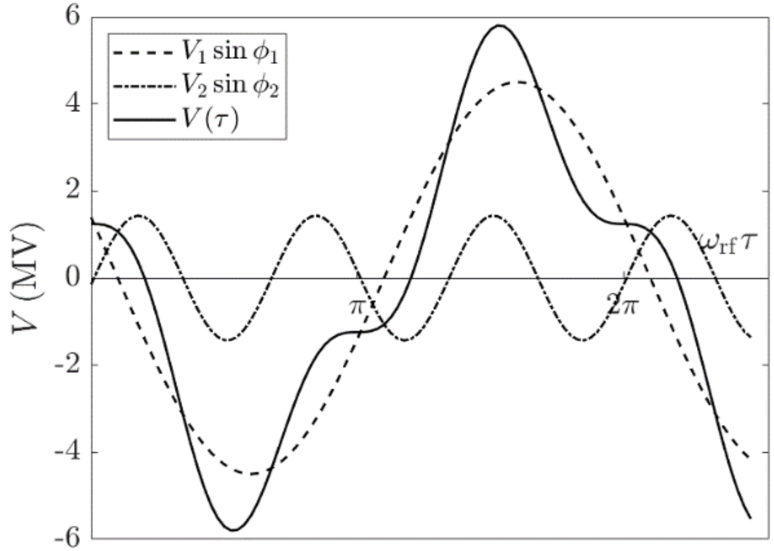


Higher Harmonic Cavity and Bunch Lengthening



Bunch lengthening factor

$$u = \sqrt{\frac{\Gamma\left(\frac{3}{4}\right)}{\Gamma\left(\frac{1}{4}\right)}} \left[\left(\frac{Q_s}{h\eta}\right)^2 \frac{24}{m^2-1} \frac{\cos\phi_{0s}}{\cos\phi_{1s}}\right]^{\frac{1}{4}} \frac{1}{\sqrt{\sigma_\delta}} \,.$$

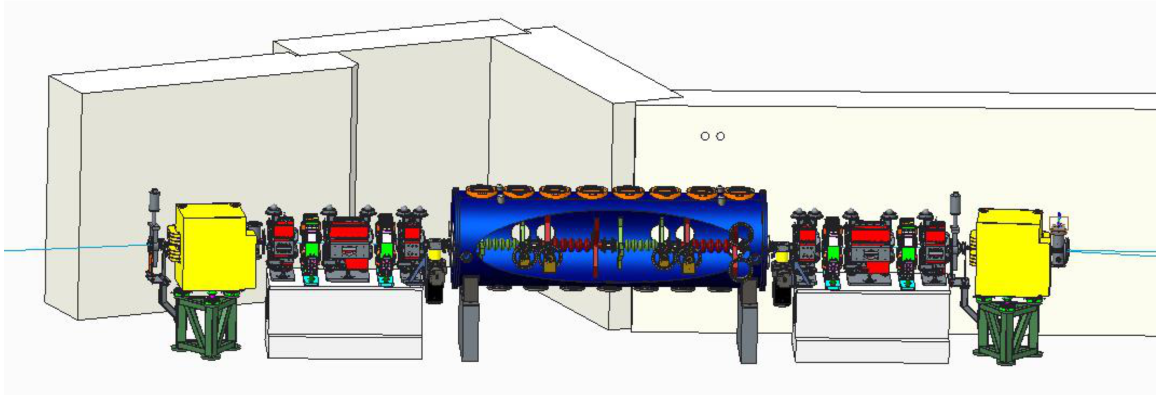
BESSY VSR – Upgrade of BESSY II

Variable pulse length **S**torage **R**ing

- An important feature of future synchrotron sources will thus be the flexibility to tune the beam parameters to the various user needs, while ideally maintaining a high average flux.
- The world-wide unique feature of the BESSY VSR will be the simultaneous operation of long and short pulses, both at high bunch current, while still providing the average beam users have come to expect from 3rd generation facilities. One may then store many long bunches to provide high photon flux while only a few short-bunch buckets are populated with high charge for short-pulse experiments.
- In this alternating bunch length schema, the impedance heating effects of the machine can be avoided and the Touscheck loss rate can be limited since both effects are strongly dependent on the total currents in the short bunches and these are only a smaller fraction compared to the current in the long bunches.

BESSY VSR – Upgrade of BESSY II

- This new operating mode is achieved by installing longitudinally focusing SRF cavity systems of 1.5 GHz and 1.75 GHz in addition to NC 0.5 GHz RF system into one of the low β straights of the BESSY II ring.



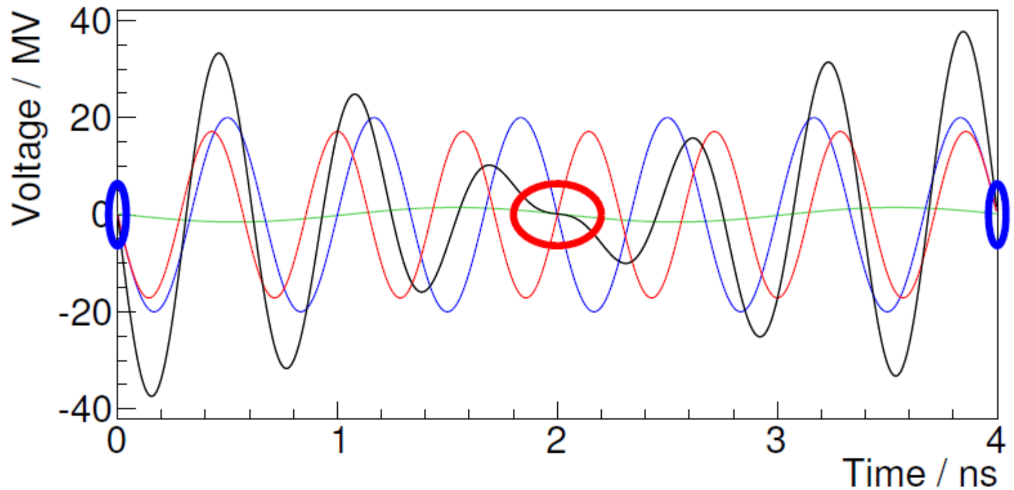
- The two different frequencies cause a beating of the voltage creating fixed points where the voltages add and others where they cancel. Short bunches are located at the high-voltage gradient points and long bunches at the cancellation points.
- The transverse beam optics and emittance remains unchanged.

BESSY VSR – Upgrade of BESSY II

: Overview of relevant BESSY II parameters.

Parameter	Value
Energy E	1.7 GeV
Emittance ϵ	5 nm rad
Coupling	2 %
Beam optics, DBA cells	2×8
Circumference	240 m
Max. beam current I	300 mA
Harmonic number h	400
RF frequency f_{rf}	500 MHz
RF sum voltage V_{rf} at 500 MHz	1.5 MV
Landau cavities frequency	1.5 GHz
Landau cavities sum voltage	0.225 MV
Revolution frequency f_{rev}	1.25 MHz
Momentum compaction factor α	7.3×10^{-4}
Relative natural energy spread, rms δ_0	7×10^{-4}
Transversal tunes Q_x, Q_y	17.85, 6.74
Synchrotron frequency f_s	7.6 kHz *
Longitudinal radiation damping time τ_z	8 ms
Transverse radiation damping time $\tau_{x,y}$	16 ms

* without Landau cavities



Facility	Peak brilliance	Average brilliance	Number of bunches	Pulse duration
	ph/s/mrad^2/mm^2/0.1 %BW	(multi bunch curr.)		ps (rms)
BESSY II				
standard	6.1e21	4e19 (300 mA)	350	15
low α	1.9e20	2.5e17 (15 mA)	350	3
BESSY VSR				
total	varying	4e19 (300 mA)	varying	varying
std. long pulses	1.2e22	3.3e19 (248 mA)	150	15
std. short pulses	1.7e22	3.6e18 (27 mA)	150	1.1
long camshaft	3.95e22	1.3e18 (10 mA)	1	27
short camshaft	5.1e22	1e17 (0.8 mA)	1	1.7
low α	2.2e21	1.2e17 (7.5 mA)	150	0.3

BESSY VSR – Upgrade of BESSY II

- The key elements of BESSY VSR are SC cavities, cooled down to 1.8 K and operating at two different frequencies and voltage amplitude.

Cavity	Frequency f /GHz	Integrated voltage V /MV	Number of cavities
NC	0.5	1.5	4
SC_1	1.5	20	2 × 5 cells
SC_2	1.75	17.14	2 × 5 cells

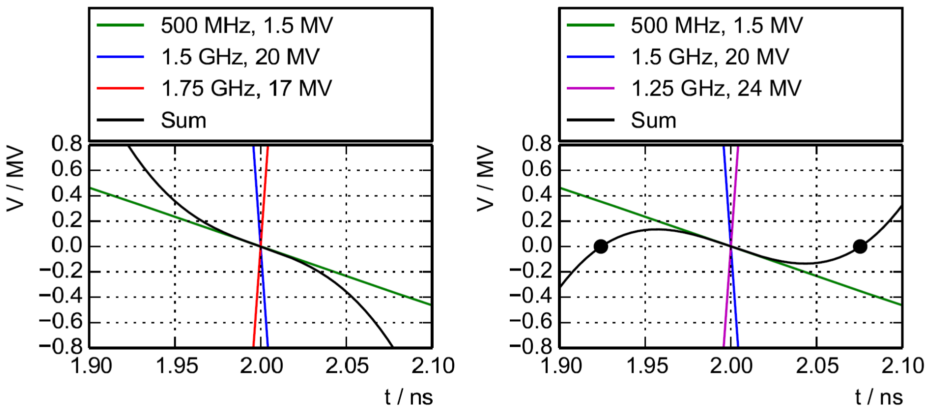


Figure 2.4: Voltages of the cavities in the close vicinity of the stable fix point of the long bunch. Left: BESSY VSR standard setup with $f_{sc,2} > f_{sc,1}$. Right: Hypothetical setup with $f_{sc,2} < f_{sc,1}$ which leads to limitation of the bucket by the additional unstable fixed points (black dots) and very low RF acceptance.

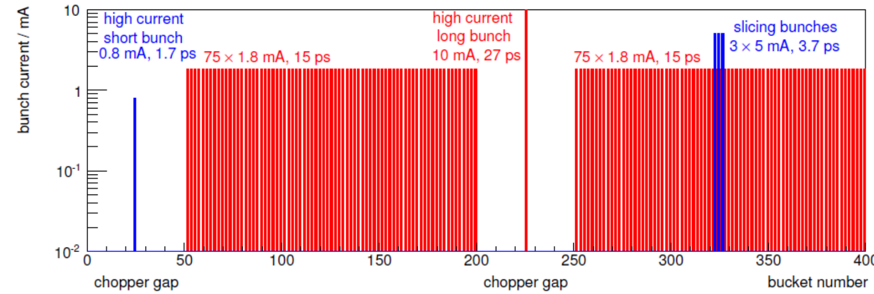


Figure 2.5: Intended fill pattern for BESSY VSR operation with short bunches (blue) and long bunches (red). Two chopper gaps are introduced to enable photon beam separation. Overall beam current is 300 mA.

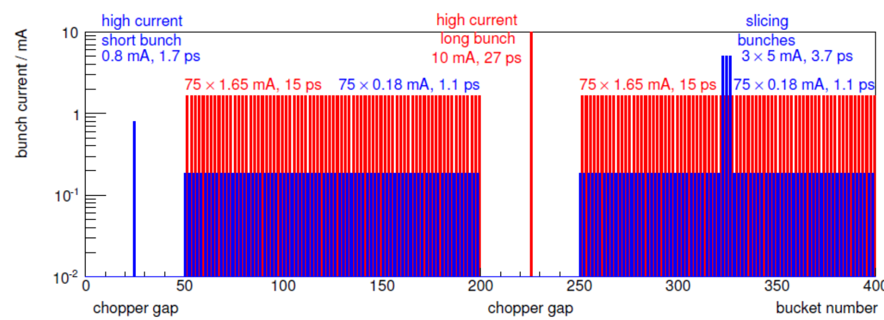


Figure 2.6: Possible fill pattern for BESSY VSR operation with short bunches (blue) and long bunches (red). Trains of short bunches are added to relax beam lifetime and to supply THz power as well as high repetition rate short x-ray pulses. Overall beam current is 300 mA.