VSR – Alternating Bunch Length Schemes

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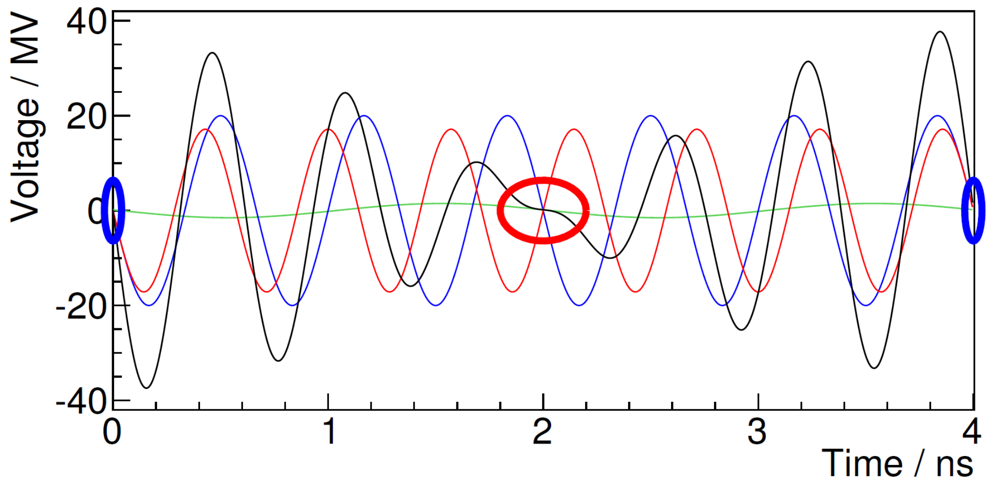
Alternating Bunch Length Scheme 1



Consider the VSR becomes a special case for dual HHC problem with $m_1 = m$ and $m_2 = m + \frac{1}{2}$.

Then the optimal bunch length condition is obatained by Flat Potential conditions:

$V(0) = 0, V'(0) = 0, V''(0) = 0, V^{(3)}(0) = 0, V^{(4)}(0) = 0$, where

$$V(\tau) = V_1 \sin(\omega_{\rm rf}\tau + \phi_{1s}) + V_2 \sin(m\omega_{\rm rf}\tau + \phi_{1s}) + V_3 \sin\left[\left(m + \frac{1}{2}\right)\omega_{\rm rf}\tau + \phi_{1s}\right] - \frac{U_0}{e} .$$

Therefore, the VSR parameters for PLS $-$ II with $m = 3$ are given as

Alternating Bunch Length Scheme 1

$$\phi_{0s} = \pi - \sin^{-1}\left(\frac{U_0}{eV}\right) = 163.98°\,, \qquad \phi_{1s} = \pi - \sin^{-1}\left(\frac{m^2\left(m+\frac{1}{2}\right)^2}{(m^2-1)\left[\left(m+\frac{1}{2}\right)^2 - 1\right]}\sin\phi_{0s}\right) = 160.24°$$

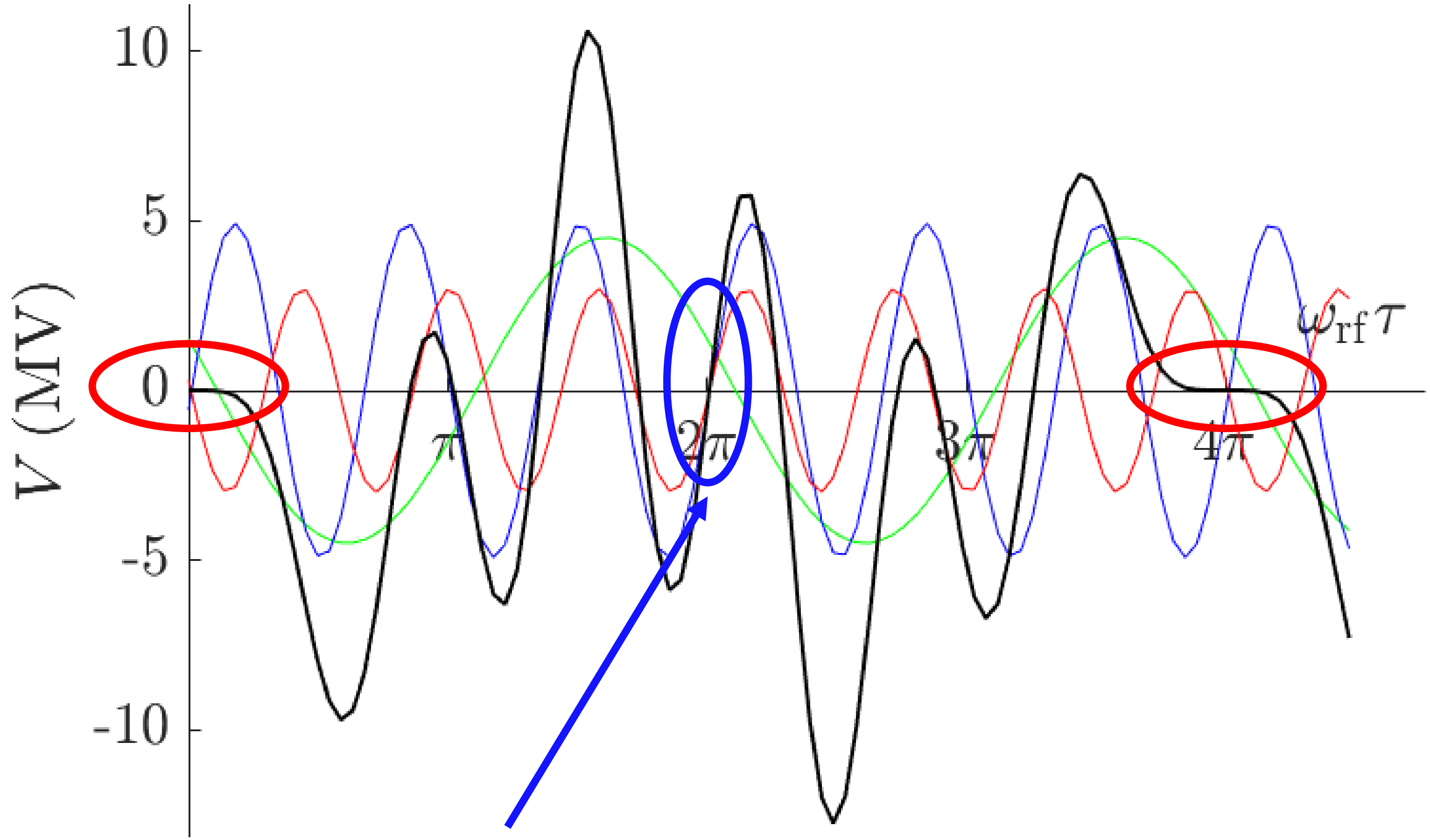
$$\phi_{2s} = \pi - \tan^{-1}\left(\frac{m\left(m+\frac{1}{2}\right)^2\sin\phi_{0s}}{\sqrt{(m^2-1)^2\left[\left(m+\frac{1}{2}\right)^2-1\right]^2 - m^4\left(m+\frac{1}{2}\right)^4\sin^2\phi_{0s}}}\right) = 173.17°$$

$$\phi_{3s} = \pi - \tan^{-1}\left(\frac{m^2\left(m+\frac{1}{2}\right)\sin\phi_{0s}}{\sqrt{(m^2-1)^2\left[\left(m+\frac{1}{2}\right)^2-1\right]^2 - m^4\left(m+\frac{1}{2}\right)^4\sin^2\phi_{0s}}}\right) = 174.14°$$

$$r_1 = \frac{1}{m}\sqrt{\frac{(m^2-1)\left[\left(m+\frac{1}{2}\right)^2-1\right]^2 - m^2\left(m+\frac{1}{2}\right)^4\sin^2\phi_{0s}}{(m^2-1)\left(m+\frac{1}{4}\right)^2}} = 1.09$$

$$r_2 = -\frac{1}{m+\frac{1}{2}}\sqrt{\frac{(m^2-1)^2\left[\left(m+\frac{1}{2}\right)^2-1\right] - m^4\left(m+\frac{1}{2}\right)^2\sin^2\phi_{0s}}{\left[\left(m+\frac{1}{2}\right)^2-1\right]\left(m+\frac{1}{4}\right)^2}} = -0.67$$

Alternating Bunch Length Scheme 1



Shortest bunch is not approved!

Important Relation about natural bunch length

Recall that the natural bunch length in length unit is given as

$$\sigma_{z0} = \frac{\eta c}{2\pi f_s}\sigma_\delta,$$

where $f_s^2 = f_0^2 \cdot \frac{heV_0\eta|\cos\phi_s|}{2\pi E}$ with $\eta > 0$ and $\beta = \frac{v}{c} \approx 1$ is assumed.

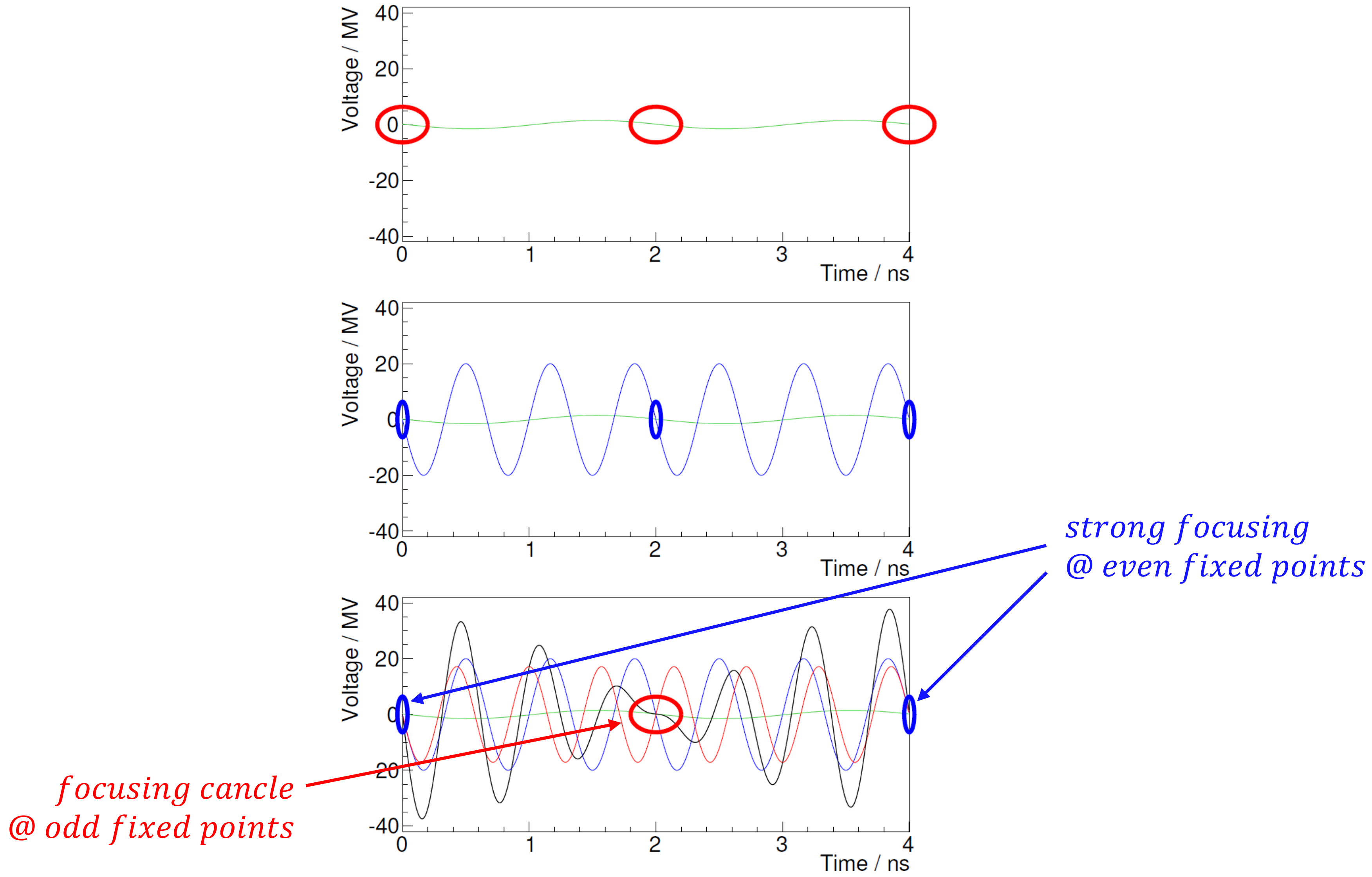
Also, for sinusoidal RF voltage, $V' = \frac{\partial V}{\partial z} = \frac{1}{c}\frac{\partial V}{\partial t} = \frac{1}{c}\cdot\omega_{\mathrm{rf}}V_0 = \frac{h\omega_0}{c}V_0 = \frac{2\pi h f_0}{c}V_0$, hence f_s^2

$= f_0 \cdot \frac{\eta eV'c|\cos\phi_s|}{4\pi^2 E}$. Therefore,

$$\sigma_{z0} \propto \sqrt{\frac{\eta}{V'}}\ .$$

E.g. For a bunch length reduction by a factor of approximately 9 to get picosecond and sub-Picosecond bunches, an 80 times stronger gradient is required. → SC cavities are required.

Alternating Bunch Length Scheme 2



Alternating Bunch Length Scheme 2

Choose $\phi_{0s} = \phi_{1s} = \phi_{2s}$ and generalize for f_1 and f_2 ;

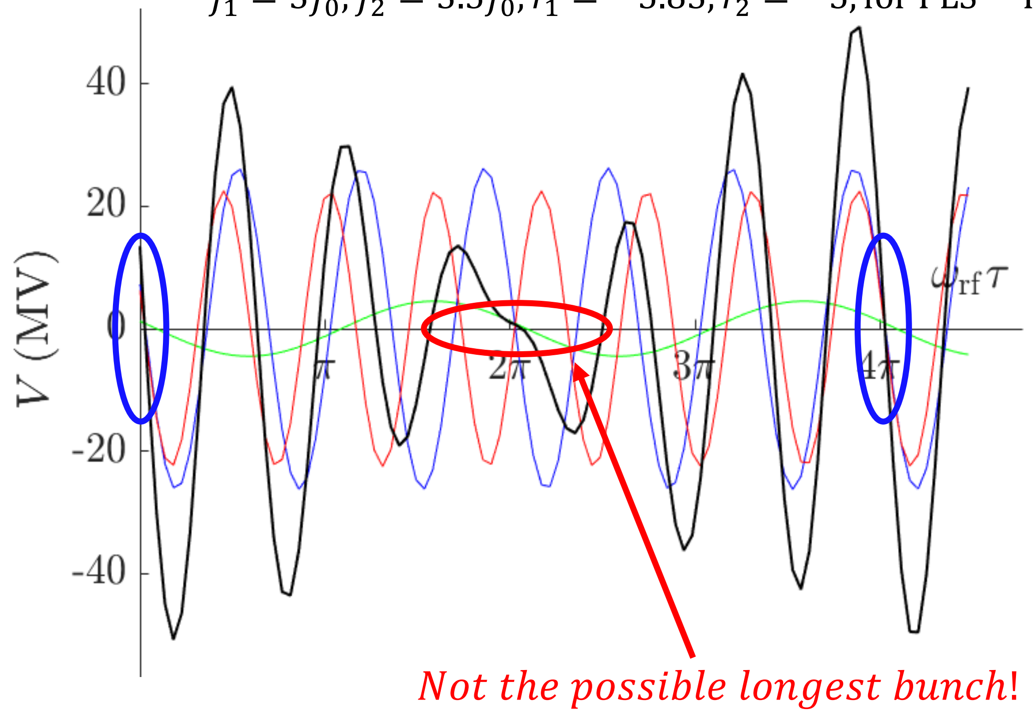
Even fixed points :

$$V' = \frac{2\pi h}{c}(f_0V_0 + f_1V_1 + f_2V_2)$$

Odd fixed points at $f_1V_1 + f_2V_2 = 0$, :

$$V' = \frac{2\pi h}{c} f_0V_0, \qquad \left|\frac{V_1}{V_2}\right| = \frac{f_2}{f_1}$$

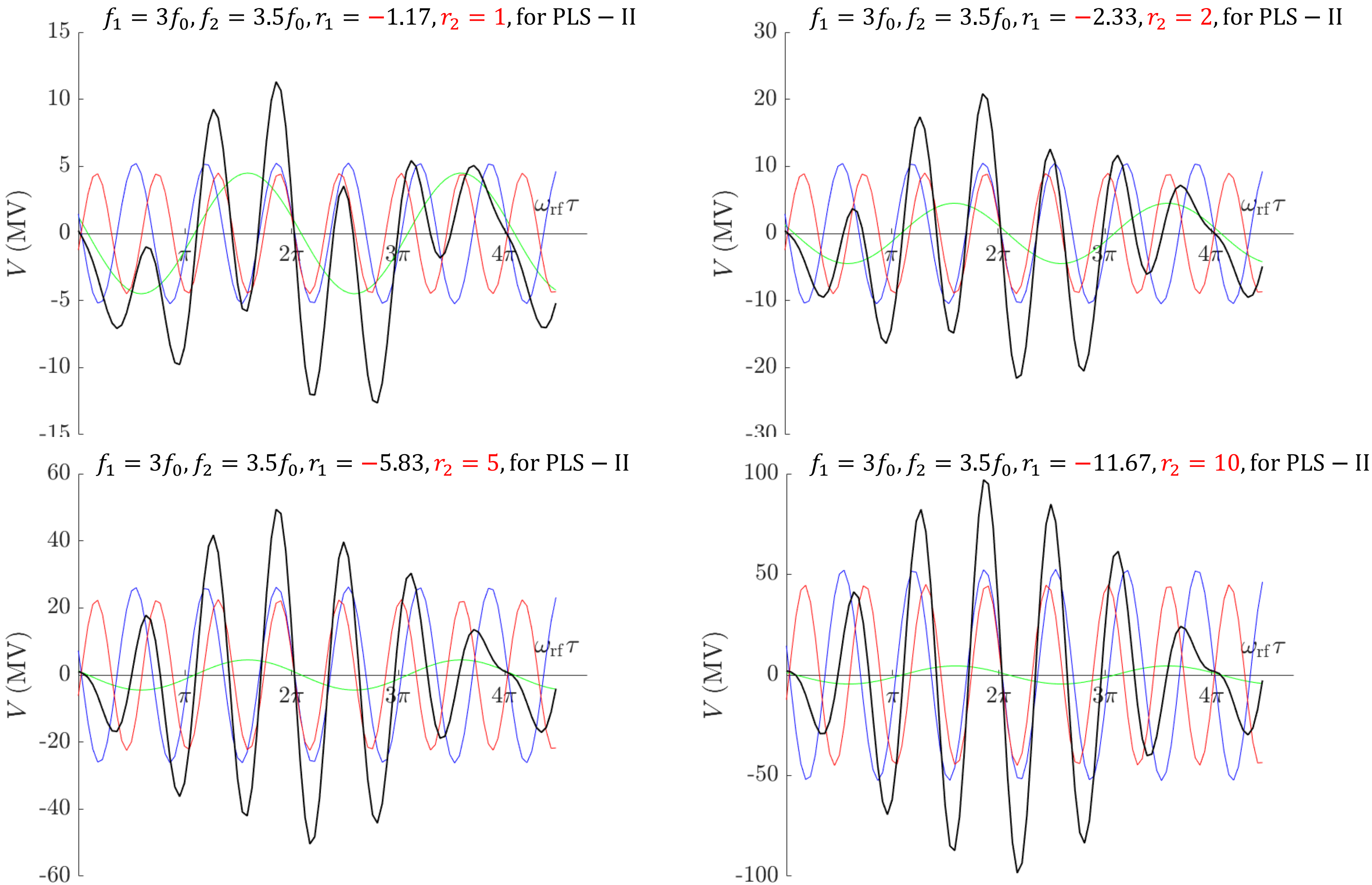
$f_1 = 3f_0, f_2 = 3.5f_0, r_1 = -5.83, r_2 = -5$, for PLS − II



To increase the gradient at even fixed points and flatten the potential at odd fixed points simultaneously, we have to choose V_1 (V_2) as high as possible.

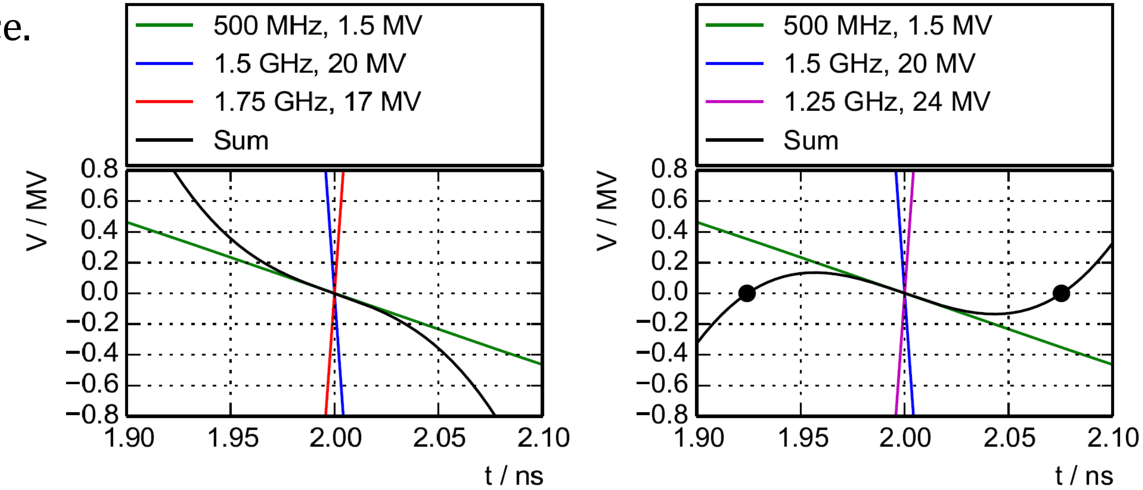
→ Not suitable for high RF voltage accelerators.

Alternating Bunch Length Scheme 2



Choices of frequencies

- If $f_2 < f_1$, the bucket is limited by the additional unstable fixed points and leads to very low RF acceptance.



- If $f_1 = 2f_0$ and $f_2 = 2.5f_0$, the cavities become larger at the same V' and might not longer fit into one straight section or would require operation at an unreasonablt high accelerating field.

- If $f_1 = 4f_0$ and $f_2 = 4.5f_0$, the structure become smaller and raises the concern of increased excitation of HOMs, leading to beam instabilities.

- Technical aspects, e.g. cryogenic losses, availability, scalability and maturity of presently available SC multi-cell cavities and RF generators favor the choice of the parameters.

More advanced steps [1]

- We can combine the alternating bunch length scheme with a low $- \alpha$ optics to go to even shorter bunches.
- To avoid bunch lengthening of ultra short bunches by coupling effects, zero dispersion is required at the cavity location.
- A two bucket scheme-displaced by few percent in energy-can be applied in low $- \alpha$ optics.
- The short bunch can be placed in the high energy bucket and the long bunch can be placed in the low energy bucket, separated by dispersive orbits.
- The emitted X-rays can be spatial separated and users can choose at the beam port between long and short X-ray pulses.

[1] Simultaneous long and short electron bunches in the BESSY II storage ring, G. Wüstefeld, *et. al.*, *Proceedings of IPAC2011, San Sebastián, Spain*